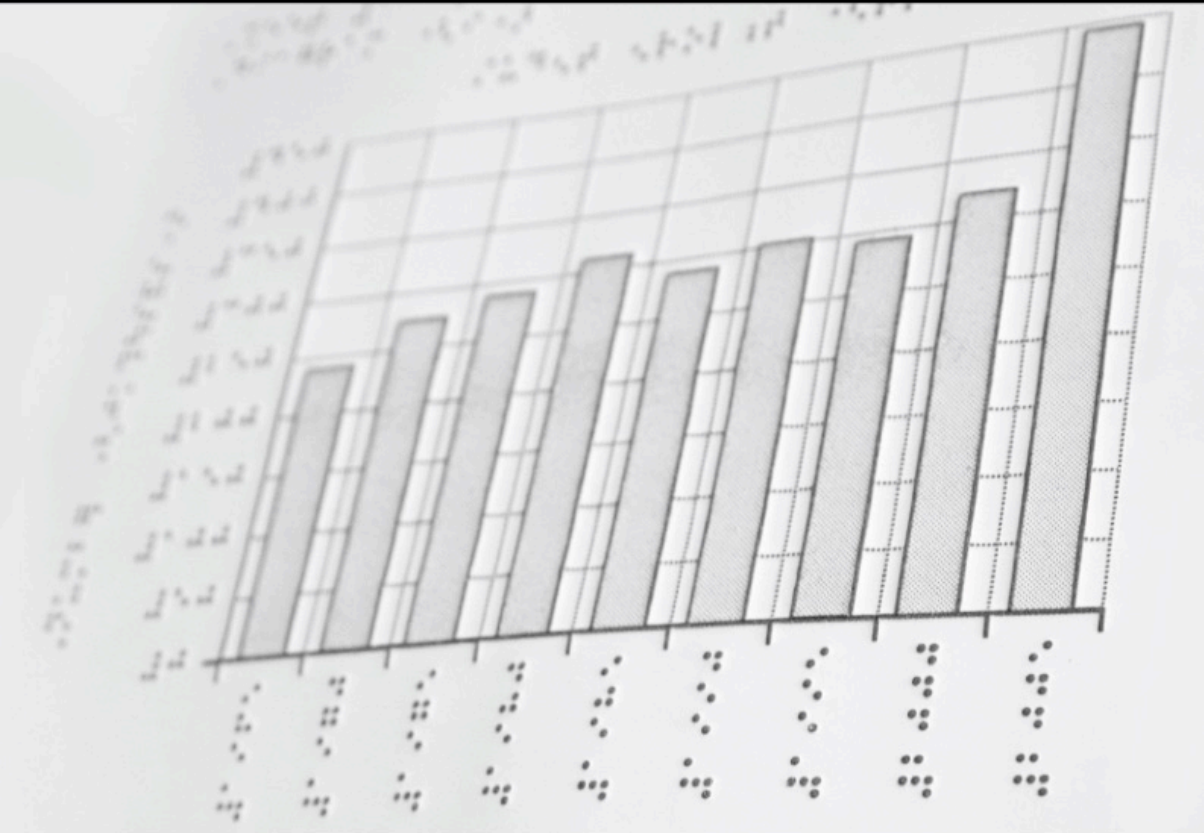
Next Sense

Advancing the RIDBC Renwick Centre experience.

Unified English Braille Training Manual

Advanced Mathematics

Josie Howse



UEBOnline

Unified English Braille Training Manual: Advanced Mathematics
Revision 7

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NextSense Institute is Australia's leading centre for research and professional studies in the field of education for children with sensory disabilities; offering webinars, short courses, and degree programs for parents, carers, educators, and health professionals. NextSense Institute is committed to providing high-quality teaching and learning opportunities. Our programs are conducted by leading national and international experts for education and health professionals who support people who are deaf, hard of hearing, blind or have low vision.

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Foreword

If you are reading this Foreword, it is most likely that you have successfully completed the online Unified English Braille (UEB) Introductory Mathematics training program. Please accept my congratulations for your success to date!

The **purpose** of this UEB Advanced Mathematics training program is to provide instruction in the reading and writing of secondary-level mathematical symbols in Unified English Braille. Mathematics is widely considered a core learning area and an essential requirement for the study of the STEM subjects of Science, Technology and Engineering. Mathematics teachers generally possess specialised mathematical knowledge and skills that enable them to effectively teach secondary students with a diverse range of abilities and attributes. However, students who use braille to access and communicate information require instruction from teachers who understand the braille code and are able to effectively modify print-based information into tactile form.

The **target audience** for UEB Advanced Mathematics includes mathematics teachers, teachers of braille, parents and caregivers, allied health professionals, education administrators and policymakers.

The **aims** of the UEB Advanced Mathematics training program are:

- To promote the acquisition of knowledge of Unified English Braille as it is applied to the continuum of those mathematical symbols that are typically taught during the secondary years of schooling; and
- To raise awareness of the enormous potential of braille knowledge and skills in enabling students with vision impairment to effectively access and engage with mathematics content and to communicate their mathematical understanding in a broad range of contexts.

The **instructional content** is presented as a series of lessons that address specific topics in secondary education. The lessons include practice and review exercises involving print to braille transcription. The content of each lesson builds on prior content, enabling the progressive development and consolidation of braille knowledge.

Please **note** that an additional UEB Online training program called UEB Extension Mathematics addresses mathematics content that is encountered during the senior years of secondary mathematics – see https://uebonline.org.

The **recommended UEB Online study sequence** is: (i) completion of UEB Literacy modules 1 and 2; followed by (ii) UEB Introductory Mathematics; and then (iii) UEB Advanced Mathematics and (iv) UEB Extension Mathematics. This step-by-step sequence is recommended because Unified English Braille is a single code system that encompasses the braille symbols for literary and technical information. It is

important, therefore, to be knowledgeable of the UEB symbols used in literary contexts, as these symbols are used for literary content of mathematical information.

On behalf of NextSense Institute, I extend my sincere thanks to those organisations and individuals who have contributed to developing the UEB Online mathematics training programs and supporting materials – see https://uebonline.org. This includes thanks to my Project Team colleagues, Josie Howse (content author), Craig Cashmore of PeppaCode Pty Ltd (online program developer); and NextSense's Lena Karam, Sonali Marathe and Tarna Cosgrove who have assured the accuracy and accessibility of the information presented. Sincere appreciation is also extended to the Duchen Family Foundation, the JLDJS Foundation, Sibley Endowment, the Skipper-Jacobs Charitable Trust and the Thomas Hare Investments Trust. Without their financial support, the UEB Online Mathematics training programs would not have been possible.

We hope that this contribution to the disability field will provide professionals and parents with the required knowledge to support and encourage secondary school students with vision impairment to actively and confidently engage with the exciting world of mathematics.

Frances Gentle, AO PhD
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NextSense Institute

Contributors

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Josie Howse is an Adjunct Research Fellow with the NextSense Institute

and former Manager of the Braille and Large Print Services, NSW Department of Education. The NSW Department of Education team provides all texts and examinations in braille, large print and e-text to students with vision impairment in the government sector and is the largest producer of alternative format student textbooks and examinations in Australia.

Josie has been working in the field of vision impairment for more than 40 years. She has held a number of executive positions at national and state levels and has extensive experience in braille code development at an international level. Josie was the editor of the 2006 Unified English Braille Primer: Australian Edition, and co-editor of the 2016 Unified English Braille: Australian Training Manual. Josie was awarded the Public Service Medal (PSM) in the Queen's Birthday Honours list in 2007, has been listed annually in Who's Who of Australian Women since 2008, and is the recipient of a Lifetime Achievement Award from the Round Table on Information Access for People with Print Disabilities in 2012.

UEB Online Developer: Craig Cashmore
B. Eng. (Hons) – UTS

Craig holds an Engineering Degree in Telecommunications and has worked in the software development industry for over 30 years, holding senior software design, software architecture and technical management positions in companies including Jtec, Ericsson and LongReach Networks.

More recently Craig founded PeppaCode, a web and app development business focused on 'out-of-the-ordinary' strategic web and software development for small business, start-ups and educational institutions.

Some of Craig's achievements at PeppaCode include the successful launch of UEB Online for NextSense and a vehicle tracking and management system for a bus operator. Craig continues to work on new and innovative projects using modern web technologies.

Advanced Mathematics

Lesson 1: Grade 1 Mode (Symbol Indicator) and Algebra

Grade 1 Mode

The rules for the use of Grade 1 mode in a literary context will also apply in a mathematical context. Please revise the information presented in Lesson 3 of the *Unified English Braille Training Manual: Introductory Mathematics*.

A braille symbol may have both an uncontracted (Grade 1) meaning and a contracted (Grade 2) meaning. A Grade 1 indicator is used to set Grade 1 mode when the Grade 1 meaning of a symbol could be misread as a contraction meaning or a numeric meaning. The extent of Grade 1 mode is determined by the Grade 1 indicator in use.

⠰ Grade 1 Symbol indicator (dots 56)

The following Grade 1 mode indicators will be introduced more fully in Lesson 2 of this Training Manual.

⠰⠰	Grade 1 Word Indicator
⠰⠰⠰	Grade 1 Passage Indicator
⠰⠄	Grade 1 Passage Terminator
⠐⠐⠿⠰⠰⠰	Grade 1 Passage Indicator on a line of its own
⠐⠐⠿⠰⠄	Grade 1 Passage Terminator on a line of its own

Note:

Grade 1 indicators will not be needed for simple arithmetic problems involving number, operation signs, numerical fractions and mixed numbers.

Grade 1 Symbol Indicator

- A Grade 1 symbol indicator (⠰) sets Grade 1 mode for only the next symbol.

Example:

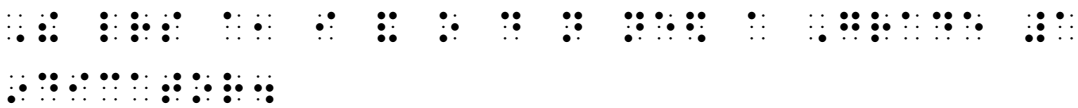
Seat 10a

⠠⠎⠂⠞ ⠼⠁⠚⠰⠁

- A Grade 1 symbol indicator is NOT required before the letters a, i and o, because they do not have a contraction meaning when they “stand alone”.

Example:

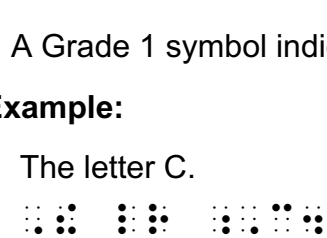
The letters a, i and o do not need a Grade 1 indicator.



- A Grade 1 symbol indicator precedes a capitalisation indicator.

Example:

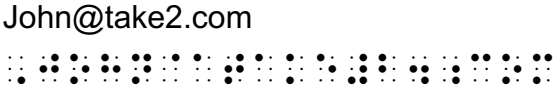
The letter C.



- A Grade 1 symbol indicator is required to prevent a letter from being misread as a number.

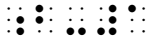
Example:

John@take2.com



- A Grade 1 symbol indicator may be required for a sequence of letters in braille that could represent an alphabetic wordsign, or a shortform such as ab (about) or ac (according), if the letter or letter sequence is “standing alone”.

Examples:

b-1 

B-1

ab = ac

Numeric Indicators Set Grade 1 Mode

The numeric mode indicator also sets Grade 1 mode for the remainder of the symbols-sequence.

Examples:

4th

1st

geeks2you

4starhotel@webnet.com

When Grade 1 mode is set by a numeric indicator, it is terminated by a space, a hyphen, a dash or a Grade 1 terminator. More information on Grade 1 mode will be included in Lesson 2.

Examples:

3-D ⠼⠉⠤⠰⠠⠙

Remember, the Numeric mode is terminated by a space or any symbol **not** listed below:

- The ten digits
- Full stop (period)
- Comma;
- The numeric space
- Simple numeric fraction line; and
- The continuation indicator

Therefore, the numeric mode is terminated by symbols such as the hyphen, dash, slash/oblique stroke and colon.

Examples:

1939-45
⠼⠁⠊⠉⠊⠤⠼⠙⠑

1939–1945
⠼⠁⠊⠉⠊⠠⠤⠼⠁⠊⠙⠑

24/7
⠼⠃⠙⠸⠌⠼⠛

8:30 am
⠼⠓⠒⠼⠉⠚ ⠁⠍

Standing Alone Rule

Refer to *The Rules of Unified English Braille* (Simpson, 2013) for a comprehensive definition and explanation of usage of “standing alone”.

A letter or letter sequence is considered “standing alone” when:

- it is preceded and followed by a space, a hyphen or a dash (of any length); or
- common punctuation and indicator symbols intervene between the letter or letters-sequence and the **preceding** or **following** space, hyphen or dash or
- a word with an interior apostrophe is considered to be “standing alone” under the specific provisions referring to contractions, alphabetic wordsigns, strong wordsigns and shortforms.

Examples:

(c

⠐⠣⠰⠉

(C

⠐⠣⠰⠠⠉

“can

⠦⠉

“p’s and q’s”

⠦⠰⠏⠄⠎ ⠯ ⠰⠟⠄⠎⠴

General Note

Decisions often need to be made in a mathematical context about whether to use:

- a Grade 1 symbol indicator or
- a Grade 1 word indicator or
- a Grade 1 passage indicator with a Grade 1 terminator or
- a Grade 1 passage indicator on a line of its own with a Grade 1 passage terminator on a line of its own.

Often there is a choice about Grade 1 indicators in mathematical contexts, with any of the options (above) being equally correct. Decisions about option selection are generally associated with user and transcriber preferences, including consideration for simplicity or functionality.

Algebra

- Algebra is the part of mathematics in which letters and other general symbols are used to represent numbers and quantities in formulae and equations.
- Simple algebraic equations which include letters may need Grade 1 symbol indicators where the letters stand alone, or the letters a-j immediately follow the numbers and the letters themselves may be misread as numbers.
- The ratio sign (⠒) has a Grade 2 (contracted) meaning, so unless you are already in Grade 1 mode, a Grade 1 symbol indicator will be required before the ratio sign.
- Remember that the numeric indicator also sets Grade 1 mode for the next symbols-sequence. When Grade 1 mode is set by a numeric indicator, it is terminated by a space, a hyphen, a dash or a Grade 1 terminator.

Notes:

1. The presentation of algebraic expressions in print is often shown in italics. This is generally ignored in braille.
2. If the braille representation for a print sequence in the following exercises does not fit on the line, then the first preference would be to break:

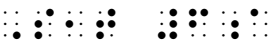
- before comparison signs,
- before operation signs, or
- before a mathematical unit such as
    - fractions
    - functions
    - radicals
    - items with modifiers such as superscripts or bars
    - shapes or arrows
    - anything enclosed in print or braille grouping symbols.

Usually the best place to break is before a comparison sign or an operation sign.

Examples:

$x = y + 5c$

Seat 6a



The ratio of $x : y$

The ratio $5 : 10$

Exercise 1

1. $a + b = c$
2. $x + y = z$
3. $2x = y$
4. $am - an$
5. x-axis
6. X-axis
7. The product of a and b, (ab), gives the area.
8. The ratio $p: q$
9. $Ab = Xb + Yb$
10. $4r \div 2r =$

Extra Exercise 1

1. Expand $(a + 2)(a + 4)$
2. Expand and simplify $(3a + 5)(a + 1)$
3. $(q + 4)(q - 2)$
4. $2x(3x - 1) + 3(3x - 1)$
5. $ab + cd = gd$
6. $(m + 3)(m + 3) = m(m + 3) + 3(m + 3)$
7. $21a \div 7a = 3$
8. $mn + yz =$
9. $7b + 5b =$
10. $(t - 4)(t - 7)$

Lesson 2: Grade 1 Mode (Word and Passage) and Fractions (continued)

Grade 1 Mode

The rules for the use of Grade 1 mode in a literary context will also apply in a mathematical context. Please revise the information presented in Lesson 3 of the *Unified English Braille Training Manual: Introductory Mathematics* and Lesson 1 of this Training Manual.

A braille symbol may have both an uncontracted (Grade 1) meaning and a contracted (Grade 2) meaning. A Grade 1 indicator is used to set Grade 1 mode when the Grade 1 meaning of a symbol could be misread as a contraction meaning or a numeric meaning. The extent of Grade 1 mode is determined by the Grade 1 indicator in use.

⠰	Grade 1 Symbol Indicator (dots 56)
⠰⠰	Grade 1 Word Indicator
⠰⠰⠰	Grade 1 Passage Indicator
⠰⠄	Grade 1 Passage Terminator
⠠⠠⠿⠰⠰⠰	Grade 1 Passage Indicator on a line of its own
⠠⠠⠿⠰⠄	Grade 1 Passage Terminator on a line of its own

Note:

Grade 1 indicators will not be needed for simple arithmetic problems involving number, operation signs, numerical fractions and mixed numbers.

Grade 1 Word Indicator

- The Grade 1 word indicator (⠰⠰) sets Grade 1 mode for the following sequence of symbols or the remainder of the current symbol sequence.
- The effect of a Grade 1 word indicator is terminated by a space or a Grade 1 terminator.
- Remember that the numeric indicator (⠼) also sets Grade 1 mode for the next symbols-sequence.
- Complex algebraic expressions that do **not** include a comparison sign are best shown using Grade 1 word mode.
- More examples of the use of Grade 1 word mode in a mathematical context will be presented later in this manual.

Note:

The example below is a General Fraction and will be dealt with in more detail in Lesson 2.

Example:

Replace I with $\frac{E}{R}$

⠠⠗⠑⠏⠇⠁⠉⠑ ⠠⠊ ⠾ ⠰⠰⠷⠠⠑⠨⠌⠠⠗⠾

Grade 1 Passage Indicator and Grade 1 Passage Terminator

- A Grade 1 passage indicator (⠰⠰⠰) sets Grade 1 mode for the next passage.
- A Grade 1 passage is terminated by the Grade 1 terminator (⠰⠄).
- Complex algebraic equations that include a comparison sign are best enclosed in Grade 1 passage indicators. This will ensure that isolated letters and indicators such as superscripts, subscripts, fractions, radicals, arrows and shapes are well defined without the need for grade symbol indicators.
- More examples of the use of Grade 1 passage mode in a mathematical context will be presented later in this manual.

Grade 1 Passage Terminator

The Grade 1 passage terminator (⠰⠄) follows immediately after the last affected symbols-sequence of a Grade 1 passage.

Example:

$x = y - z, \quad y = x + z, \quad z = y - x$

⠰⠰⠰⠭ ⠐⠶ ⠽⠐⠤⠵⠂ ⠽ ⠐⠶ ⠭⠐⠖⠵⠂ ⠵ ⠐⠶ ⠽⠐⠤⠭⠰⠄

Grade 1 Passage Indicator and Terminator on a line of their own

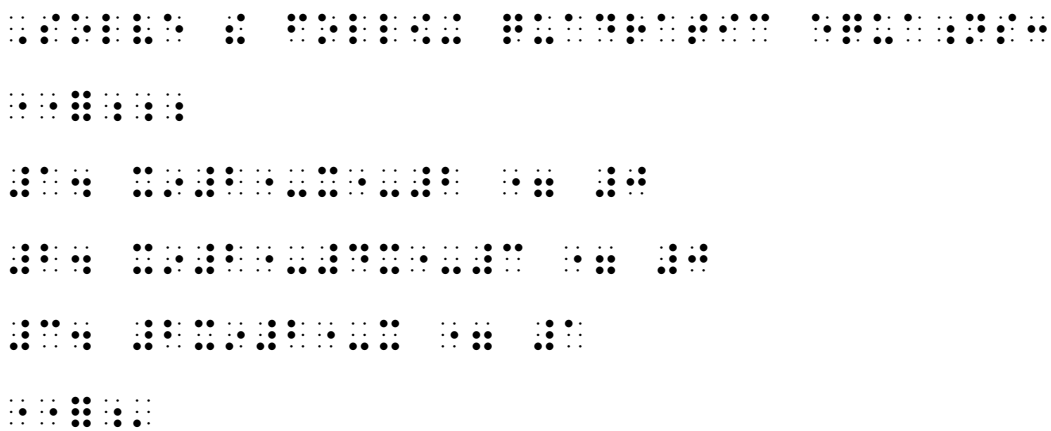
Where it is better to preserve the natural alignment of a text, for example, in a computer program or a set of equations in mathematics, place the Grade 1 passage indicator (⠰⠰⠰) on a separate line above the equations (consider the margin as a suitable location), and the Grade 1 terminator (⠰⠄) on a separate line below the equations. (Refer to the example below which shows this use with a set of quadratic equations using superscripts. Superscripts will be explained further in Lesson 4 of this Training Manual).

When the Grade 1 passage indicator and terminator is placed on a line of its own, each indicator should be preceded by the dot locator for “use” (⠐⠐⠿). A braille symbol which has only lower dots in the cell and which is isolated from other text may be misread.

Examples:

Solve the following quadratic equations:

1. $x^2 - x - 2 = 0$
2. $x^2 - 4x - 3 = 0$
3. $2x^2 - x = 1$



General Note

Decisions often need to be made in a mathematical context about whether to use:

- a Grade 1 symbol indicator or
- a Grade 1 word indicator or
- a Grade 1 passage indicator with Grade 1 terminator or
- a Grade 1 passage indicator on a line of its own with a Grade 1 passage terminator on a line of its own.

Often there is a choice about Grade 1 indicators in mathematical contexts, with any of the decision options (above) being equally correct. Decisions about option selection are generally associated with user and transcriber preferences, including consideration for simplicity or functionality.

Fractions

Lesson 8 of the *Unified English Braille Training Manual: Introductory Mathematics* introduced how to braille simple fractions, linear fractions and mixed numbers (that is, a whole number followed immediately by a simple fraction).

It is important to understand the definition of a **simple fraction**, that is, what elements and only those elements that can be considered to be a simple fraction, and therefore require use of the simple fraction line, (⠌ dots 34) between the numerator and the denominator.

If a fraction does **not** comply with the **simple numeric fraction** definition (below) for whatever reason, then it will be a **general fraction** and will require a different approach, using different signs.

Simple numeric fraction definition (Revision)

Simple numeric fractions were introduced in Lesson 8 of the *Unified English Braille Training Manual: Introductory Mathematics*.

Definition: A simple numeric fraction is one whose numerator (top of the fraction line) and denominator (bottom of the fraction line) contain only:

- digits,
- decimal points,
- commas, or
- separator spaces,
- **and**, if the fraction line in print (often referred to as the vinculum) is drawn between the two vertically (or near vertically) arranged numbers as shown in the print.

If the fraction complies fully with the definition above for a simple fraction, then a numeric fraction line symbol (⠌) should be used between the numerator and the denominator and the numeric fraction line symbol continues the numeric mode and the numeric indicator will not need to be repeated after the fraction line.

Examples:

⠼⠁⠌⠼⠃	$\frac{1}{2}$	(vertically) or
⠼⠁⠌⠼⠃	$^{1}/_{2}$	(near vertically)
⠼⠁⠊⠌⠼⠃⠁	$\frac{196}{28}$	(vertically) or
⠼⠁⠊⠌⠼⠃⠁	$^{196}/_{28}$	(near vertically)
⠼⠃⠄⠑⠚⠚⠌⠼⠁⠚⠄⠚⠚⠚	$\frac{2.500}{10.000}$	(decimals)
⠼⠁⠚⠂⠚⠚⠚⠌⠼⠑⠚⠂⠚⠚⠚	$\frac{10{,}000}{50{,}000}$	(commas)
⠼⠁⠚⠀⠚⠚⠚⠌⠼⠑⠚⠀⠚⠚⠚	$\frac{10\ 000}{50\ 000}$	(separator spaces)

Linear Fractions (Revision)

The simple numeric fraction line is not used when the print is expressed linearly, that is, where all the numbers in the print (numerator and denominator) are positioned sitting horizontally on the line, using an ordinary forward slash symbol (⠸⠌). In such instances, the same forward slash is used as a print slash and the numeric indicator will need to be repeated as the slash sign terminates numeric mode.

Example:

5/8 ⠼⠑⠸⠌⠼⠓

General Fractions

General Fraction Indicators

The braille symbols shown below for the opening and closing general fraction indicators have a contracted (Grade 2) meaning and so have been shown with a Grade 1 indicator in front of them.

A Grade 1 indicator is used to set Grade 1 mode when the Grade 1 meaning of the symbol could be misread as a contraction meaning. The extent of Grade 1 mode is determined by the Grade 1 indicator in use.

⠸⠷ Opening general fraction indicator

⠸⠾ Closing general fraction indicator

⠸⠌ General fraction line

- If the numerator or denominator of a fraction is not entirely numeric, as defined above for a simple fraction, then the general fraction indicators should be used. Write the opening general fraction indicator (⠸⠷), then the numerator (top) expression, then the general fraction line symbol (⠸⠌), then the denominator (bottom) expression and finally, the closing general fraction indicator (⠸⠾) (see examples below).
- The numerator and denominator may be any kind of expression, including fractions of either simple numeric or general type.
- Remember Grade 1 mode has been set by the numeric indicator (⠼) and is terminated by the space, hyphen, dash or Grade 1 terminator.
- The following examples of General Fractions show the choice that can be made when using the Grade 1 indicator. There is usually a preferred option.

Examples:

$$\frac{2+3}{4-1}$$

$$\frac{x+y}{x-y}$$

OR

$$\frac{a}{b}c$$

OR

$$Speed = \frac{distance}{time}$$

(the first example shown in braille below is the preferred option so as to retain more contracted words in the literary equation)

OR

OR

$$\frac{x \times y}{p \times q} = \frac{xy}{pq}$$

OR

OR

the preferred option below, due to the use of a comparison sign used in the sequences and Grade 1 indicators that are required on both sides of the equation.

⠰⠰⠰⠷⠭⠐⠦⠽⠨⠌⠏⠐⠦⠟⠾ ⠐⠶ ⠷⠭⠽⠨⠌⠏⠟⠾⠰⠄

Notes:

1. It is important to remember that the opening and closing fraction indicators each have a Grade 2 (contracted) meaning and so if Grade 1 mode has not been established for some reason, then the opening and closing fraction indicators will require Grade 1 indicators. For more information about Grade 1 mode, refer to Lesson 1 of this Training Manual and also Lesson 3 of the *Unified English Braille Training Manual: Introductory Mathematics*.
2. As referred to in the General Note in Lesson 1, there is often a choice when selecting Grade 1 indicators in mathematical contexts, with any of the Grade 1 mode options (symbol, word or passage) being equally correct. The choice of which Grade 1 indicator is associated with user and transcriber preferences, including consideration for simplicity or functionality.
3. For the purpose of the UEB Online exercises that are associated with this UEB Training Manual: Advanced Mathematics, use the following criteria for implementing Grade 1 mode:
   - Use the Grade 1 **symbol** indicator when there is only one symbol in the sequence requiring a Grade 1 indicator.
   - Use the Grade 1 **word** indicator when there are two or more symbols in the sequence requiring Grade 1 mode, except in a context whereby any literary elements will be affected and as a consequence will also be uncontracted.
   - Use Grade 1 **passage** indicator (with Grade 1 terminator) when a comparison indicator or a space is used in the sequence and Grade 1 indicators are required on both sides of the equation. Remember however, the impact of Grade 1 mode on any literary elements.

Hints:

- Questions 1-5 in Exercise 2 below are general fractions because they contain letters.
- Questions 6-10 in Exercise 2 below are general fractions because they contain something more than digits, commas, decimal points or numeric spaces.
- Remember the numeric indicator (⠼) also sets Grade 1 mode for the remainder of the symbols-sequence.
- When Grade 1 mode is set by the numeric indicator (⠼) it is terminated by a space, hyphen, dash or Grade 1 terminator.

Exercise 2

1. Evaluate $\frac{mn}{3} \div \frac{m}{n}$
2. $\frac{PQR}{XZ}$
3. $\frac{PqR}{xz}$
4. $y = \frac{x}{2}$
5. $\frac{3a}{5} + \frac{a}{5}$
6. $\frac{2^1/_2}{x+y}$
7. $\frac{8x-24y}{8}$
8. $\frac{4m+12}{3} \times \frac{6m}{m+3}$
9. $\frac{6(h+5)}{h+9}$
10. Simplify $\frac{5w+10}{5}$

Review Exercise 2

1. $\frac{4+3+2}{6-3+8}$
2. $\frac{2/3}{5}$
3. $A = \frac{h}{2}(a+b)$
4. $\frac{3(h+5)}{h+9}$
5. $\frac{5.3}{4{,}200}$
6. $\frac{\$55}{5}$
7. $\frac{24m}{3cm}$
8. $\frac{3}{10\,000}$
9. $\frac{4{,}000}{10}$
10. $\frac{\frac{x}{2}+\frac{y}{3}}{x+y}$

Lesson 3: Operation and Comparison Signs (continued)

Operation Signs

⠐⠖	+	plus (dot 5, dots 2 3 and 5)
⠐⠤	−	minus (dot 5, dots 3 and 6)
⠐⠦	×	times, a multiplication sign as shown in print as a cross (dot 5, dots 2 3 and 6)
⠐⠲	.	a running product sign, shown as a dot, is another means of showing multiplication in print (dot 5, dots 2 and 5 6)
⠐⠌	÷	divided by (dot 5, dots 3 and 4)
⠒	:	ratio sign as shown in print as a colon (dots 2 and 5)
⠨⠖	±	plus or minus (plus over minus)
⠨⠤	∓	minus or plus (minus over plus)

Notes:

1. For the purposes of the UEB Online exercises that are associated with this UEB Training Manual: Advanced Mathematics, always **unspace** the operation sign from the sequence on either side.
2. Although the ratio sign shown above is used to compare two numbers, it is best treated as an operation sign for the purposes of spacing.
3. The ratio sign terminates the effect of the numeric indicator and will therefore need to be repeated before the number that follows.

Examples:

6:12

⠼⠖⠒⠼⠁⠃

2·4·7

⠼⠃⠲⠼⠲⠼⠛

Comparison Signs

⠐⠶	=	is equal to
⠐⠶⠠⠅	≠	not equal to (i.e. a line cutting through an equals sign)
⠈⠣	<	is less than

⠈⠜	>	is greater than
⠸⠈⠣	≤	is less than or equal to
⠸⠈⠜	≥	is greater than or equal to
⠘⠔	≈	is approximately equal to (i.e. a tilde over a tilde)

Notes:

1. For the purposes of the UEB Online exercises associated with this UEB Training Manual: Advanced Mathematics, always **space** the comparison sign from the sequence on either side.
2. There are a number of print representations to show “is approximately equal to”. Refer to “*Unified English Braille Guidelines for Technical Material*” (International Council on English Braille, 2014) for equivalent braille representations. However, only the “is approximately equal to” sign shown above may be used in the UEB Online exercises associated with the mathematics training manuals.
3. A Grade 1 indicator is needed if the algebraic letter stands alone.
4. The triangle sign used in geometry is the same sign in braille as the shape indicator (shown as a triangle shape in Lesson 6 of the *Unified English Braille Training Manual: Introductory Mathematics*).
5. Refer to the rules for the use of the shape termination indicator as outlined in Lesson 6 of the *Unified English Braille Training Manual: Introductory Mathematics*.
6. The letters a-j will require a Grade 1 indicator if immediately following a number as the letters themselves will be read as part of the number.

Examples:

$a \times b = c$

⠁⠐⠦⠃ ⠐⠶ ⠰⠉

$x = \pm 3$

⠰⠭ ⠐⠶ ⠸⠖⠼⠉

$x = \mp 3$

⠰⠭ ⠐⠶ ⠸⠤⠼⠉

$4x + 5x \neq 8x$

⠼⠙⠭⠐⠖⠼⠑⠭ ⠐⠶⠈⠱ ⠼⠓⠭

$10 < 20 < 30$

⠼⠁⠚ ⠈⠣ ⠼⠃⠚ ⠈⠣ ⠼⠉⠚

$30 > 20 > 10$

⠼⠉⠚ ⠈⠜ ⠼⠃⠚ ⠈⠜ ⠼⠁⠚

$-2 \leq x \leq 10$

⠐⠤⠼⠃ ⠸⠈⠣ ⠰⠭ ⠸⠈⠣ ⠼⠁⠚

$10 \geq x \geq -2$

⠼⠁⠚ ⠸⠈⠜ ⠰⠭ ⠸⠈⠜ ⠐⠤⠼⠃

$3.9 \times 4.1 \approx 16$

⠼⠉⠲⠊⠐⠦⠼⠙⠲⠁ ⠘⠔ ⠼⠁⠋

Remember:

As referred to in the General Note in Lesson 1 of this Training Manual, there is often a choice when selecting Grade 1 indicators in mathematical contexts, with any of the options of Grade 1 mode (symbol, word or passage) being equally correct. The choice is associated more with user and transcriber preferences, including consideration for simplicity or functionality.

For the purpose of the exercises in UEB Online that are associated with this UEB Mathematics training program, use the following criteria for implementing Grade 1 mode:

- Use the Grade 1 **symbol** indicator when there is only one symbol in the sequence requiring a Grade 1 indicator.
- Use the Grade 1 **word** indicator when there are two or more symbols in the sequence requiring Grade 1 mode except in a context whereby any literary elements will be affected and as a consequence will also be uncontracted.
- Use Grade 1 **passage** indicator (with Grade 1 terminator) when a comparison indicator or a space is used in the sequence and Grade 1 indicators are required on both sides of the equation. Remember however, the impact of Grade 1 mode on any literary elements.

Exercise 3

1. $16 - 5 \geq 10$
2. $3 + 3 \leq 3 \times 3$
3. $15 < 17$
4. $29 > 23$
5. $a + b \neq c$
6. $15xy \div 3x = \frac{15xy}{3x}$
7. $3(4 - 2x) \geq 18$
8. $2.5 \times 7.7 \approx 19$
9. $x = \pm 15$
10. $3a + 2b = 5c$

Review Exercise 3

1. $5 + -2 =$
2. $9 + -6 = +3$
3. $.672$
4. $\frac{1\ 745\ 711}{6\ 527}$
5. $\frac{\$55.50}{5}$
6. $\frac{20m}{4cm}$
7. $\frac{a}{b+c}$
8. $\frac{x}{4{,}000}$
9. Expand $(2x - 5)(3x + 7)$
10. $\frac{\frac{p}{5}+\frac{q}{10}}{p+q}$

Lesson 4: Indices

Superscripts and Subscripts

A **superscript** is a distinguishing symbol (such as a numeral or letter) that is written immediately above, OR above and to the right or left of another character.

A **subscript** is a distinguishing symbol (such as a numeral or letter) that is written immediately below, OR below and to the right or left of another character.

Level Change Indicator

The superscript and subscript level change indicators and the braille grouping indicators shown below also have a contracted (Grade 2) meaning and so have been shown with a Grade 1 indicator in front of them. A Grade 1 indicator is used to set Grade 1 mode when the Grade 1 meaning of the symbol could be misread as a contraction meaning.

⠰⠔ Level change up

⠰⠢ Level change down

⠨⠔ Expression directly above (this symbol will be introduced in more detail in the UEB Training Manual: Extension Mathematics).

⠨⠢ Expression directly below (this symbol will be introduced in more detail in UEB Training Manual: Extension Mathematics).

Braille Grouping Indicator

⠰⠣ Open braille grouping indicator (there is no print representation for this symbol in braille)

⠰⠜ Closing braille grouping indicator (there is no print representation for this symbol in braille)

Note:

When both a left-hand subscript and superscript are shown together in print (such as in atomic mass numbers), they are brailled with the subscript first followed by the superscript.

Examples:

x^n

⠭⠰⠔⠝

x_n

⠭⠰⠢⠝

$^{-}2 + ^{-}3 = ^{-}5$

⠰⠔⠐⠤⠼⠃⠐⠖⠔⠐⠤⠼⠉ ⠐⠶ ⠰⠔⠐⠤⠼⠑

OR

⠰⠰⠰⠔⠐⠤⠼⠃⠐⠖⠔⠐⠤⠼⠉ ⠐⠶ ⠔⠐⠤⠼⠑⠰⠄

$^{238}_{92}U$

⠰⠢⠼⠊⠃⠔⠼⠃⠉⠓⠠⠥

Definition of an Item

An “item” is defined as any single symbol(s) that follows immediately after the level change indicator. It is therefore important to make clear to the reader exactly what symbol(s) will be included as a consequence of the level change indicator.

There is a defined list, shown below, of the specific symbols considered to be the “next item”:

1. An entire number, i.e. the initiating numeric indicator and all succeeding symbols within the numeric mode (which would include any decimal points, commas, numeric separator spaces, or simple numeric fraction lines).
2. An entire general fraction, enclosed in general fraction indicators (⠷) and (⠾) as shown in Lesson 2 of this module. Note that Grade 1 indicators will be required if the sequence is not already in Grade 1 mode.
3. An entire radical expression, enclosed in radical indicators.
4. An arrow (shown in UEB Training Manual: Extension Mathematics).
5. An arbitrary shape.
6. Any expression enclosed in matching pairs of round parentheses, square brackets or curly braces.

If none of the above conditions apply, then the “item” is only the next symbol and may require braille grouping indicators (⠣) opening and (⠜) closing to ensure the whole of the superscript or subscript has been captured. Note that Grade 1 indicators will be required if the sequence is not already in Grade 1 mode.

Any expression enclosed in the braille grouping indicators shown above will subsequently make it clear to the reader exactly what symbols are included as part of the level change and considered to be the “next item”.

Examples:

2^2

⠼⠃⠔⠼⠃

$x^{2.2}$

⠭⠰⠔⠼⠃⠲⠃

$x^{1/3}$

⠭⠰⠔⠼⠁⠌⠉

Footnote^{56}

⠠⠋⠕⠕⠞⠝⠕⠞⠑⠰⠔⠼⠑⠋

$x^{(-1)}$

⠭⠰⠔⠐⠣⠐⠤⠼⠁⠐⠜

P_1 and P_2

⠠⠏⠰⠢⠼⠁ ⠯ ⠠⠏⠰⠢⠼⠃

H_2O

⠠⠓⠰⠢⠼⠃⠠⠕

x^{2y}

⠭⠰⠔⠰⠣⠼⠃⠽⠜

OR

⠰⠰⠭⠔⠣⠼⠃⠽⠜

Notes:

1. The rule for the definition of an item outlined above is critical to transcribing mathematics into braille correctly.
2. If none of the groupings listed above under “Definition of an Item” apply to the expression, then the item is **only** the next individual symbol.
3. Use the opening (⠣) and closing (⠜) grouping indicator (also shown above) to capture the whole of the print sequence if the definition of an item is not met.
4. Remember the numeric indicator (⠼) also sets Grade 1 mode for the remainder of the symbols-sequence.
5. When Grade 1 mode is set by the numeric indicator (⠼), it is terminated by a space, hyphen, dash or Grade 1 terminator.

Examples:

x^2

⠭⠰⠔⠼⠃

x^{-2}

⠰⠰⠭⠔⠣⠐⠤⠼⠃⠜

OR

⠭⠰⠔⠰⠣⠐⠤⠼⠃⠜

Hint:

- The superscript and subscript signs each have a Grade 2 (contracted) meaning so will always require Grade 1 indicators if the sequence is not already shown in Grade 1 mode. For more information about Grade 1 mode, refer to Lesson 1 and Lesson 2 of this Training Manual and also Lesson 3 of the *Unified English Braille Training Manual: Introductory Mathematics*.

Examples:

y^n

⠽⠰⠔⠝

y_n

⠽⠰⠢⠝

x^{-n}

⠰⠰⠭⠔⠣⠐⠤⠝⠜

OR

⠭⠰⠔⠰⠣⠐⠤⠝⠰⠜

x_{-n}

⠰⠰⠭⠢⠣⠐⠤⠝⠜

OR

⠭⠰⠢⠰⠣⠐⠤⠝⠰⠜

a^{2n}

⠰⠰⠁⠔⠣⠼⠃⠝⠜

OR

⠁⠰⠔⠰⠣⠼⠃⠝⠜

x_{2n}

⠰⠰⠭⠢⠣⠼⠃⠝⠜

OR

⠭⠰⠢⠰⠣⠼⠃⠝⠜

Remember:

As referred to in the General Note in Lesson 1 of this Training Manual, there is often a choice when selecting Grade 1 indicators in mathematical contexts, with any of the options for Grade 1 mode (symbol, word or passage) being equally correct.

However, for the purpose of the UEB Online exercises that are associated with this UEB Training Manual: Advanced Mathematics, use the following criteria for implementing Grade 1 mode:

- Use the Grade 1 **symbol** indicator when there is only one symbol in the sequence requiring a Grade 1 indicator.
- Use the Grade 1 **word** indicator when there are two or more symbols in the sequence requiring Grade 1 mode except in a context whereby any literary elements will be affected and as a consequence will also be uncontracted.
- Use Grade 1 **passage** indicator (with Grade 1 terminator) when a comparison indicator or a space is used in the sequence and Grade 1 indicators are required on both sides of the equation. Remember however, the impact of Grade 1 mode on any literary elements.

Exercise 4

1. 10^2
2. x^{b+1}
3. $x^{\frac{1}{2}}$
4. $x^{1/2}$
5. x_4
6. x_{2+}
7. x^2
8. P_x
9. $a^{-n} = \frac{1}{a^n}$
10. $a^n\, a^m = a^{n+m}$

Review Exercise 4

1. $0.0025 = 2.5 \times 10^{-3}$
2. x_{a+b}
3. $a_2 + b_3$
4. $x^y z$
5. $\frac{x^{-9}}{4}$
6. $\frac{x^y}{z}$
7. $\frac{x^{1.2}}{4}$
8. $\frac{x^{-\frac{3}{4}}}{4}$
9. $7e_{2x}$
10. $e^{x^3} y$

Lesson 5: Roots and other Radicals

The opening and closing radical signs shown below also have a contracted (Grade 2) meaning and so have been shown with a Grade 1 indicator in front of them. A Grade 1 indicator is used to set Grade 1 mode when the Grade 1 meaning of the symbol could be misread as a Grade 2 (contraction) meaning and Grade 1 mode has not already been established in the sequence.

⠰⠩	$\sqrt{\ }$	Open radical (root) sign, with vinculum
⠰⠬		Close radical (root) sign (there is no print representation for this braille symbol)
⠐⠩	√	Square root sign without a vinculum

Square Roots

The expression inside the square root sign in print (known as the **radicand**) should be preceded by the open radical (root) sign and followed by the close radical (root) sign. The radicand sign may be any expression whatsoever and may therefore contain radicals as well as other mathematical structures.

A **vinculum** is a horizontal line used in mathematical notation for a specific purpose.

In the examples below, the vinculum is used as part of the notation of the radical to highlight the radicand whose root is being indicated.

Examples:

$\sqrt{16} = 4$

⠰⠩⠼⠁⠋⠬ ⠐⠶ ⠼⠙

$\sqrt{x + y}$ (square root of x + y)

⠰⠰⠩⠭⠐⠖⠽⠬

OR

⠰⠩⠭⠐⠖⠽⠰⠬

Radical Index (Cube Roots etc)

The radical index, if present, is printed above and to the left of the radical sign. The index is shown in braille as a superscript expression immediately following the opening radical symbol.

Notes:

1. Grade 1 mode may be necessary, using a Grade 1 symbol indicator, a Grade 1 word indicator, or a Grade 1 passage indicator with Grade 1 passage terminator. Refer to Lesson 1 and Lesson 2 of this Training Manual and also Lesson 3 of the *Unified English Braille Training Manual: Introductory Mathematics* for further reference if required.
2. Ensure that whatever is showing under the root sign has been closed after the last item by using the close radical sign (⠬).
3. When Grade 1 mode is set by the numeric indicator (⠼), it is terminated by a space, hyphen, dash or Grade 1 terminator.
4. Remember the rules relating to the "next item" (as explained in Lesson 4 of this Training Manual) when using superscripts and subscripts.

Examples:

$\sqrt[3]{8} = 2$ (cubed root of 8 equals 2)

⠰⠰⠩⠔⠼⠉⠼⠓⠬ ⠐⠶ ⠼⠃

OR

⠰⠩⠰⠔⠼⠉⠼⠓⠬ ⠐⠶ ⠼⠃

$\sqrt[mn]{xy}$

⠰⠰⠩⠔⠣⠍⠝⠜⠭⠽⠬ (this is the preferred option)

OR

⠰⠩⠰⠔⠰⠣⠍⠝⠰⠜⠭⠽⠰⠬

Square Root sign on its own with no vinculum

Sometimes print omits the horizontal line (vinculum) above the radicand. A root sign showing the vinculum means exactly the same, and the use (or otherwise) of the vinculum is only a preference by publishers. A closing indicator is not required when this root sign with no vinculum is used in braille.

Notes:

1. When introducing the topic of simple roots to younger children, it is better to introduce the concept of roots with a vinculum requiring the opening and closing indicators from the beginning of their learning.
2. If, however, the square root sign is being used as an isolated graphic symbol √ then the root sign without a vinculum (⠐⠩) can be used.

3. Remember the numeric indicator (⠼) also sets Grade 1 mode for the remainder of the symbols-sequence.

Examples:

The √ sign is often shown in the text.

⠠⠮ ⠐⠩ ⠎⠊⠛⠝ ⠊⠎ ⠷⠞⠢ ⠩⠪⠝ ⠔ ⠮ ⠞⠑⠭⠞⠲

$\sqrt{16} = 4$

⠐⠩⠼⠁⠋ ⠐⠶ ⠼⠙

Remember:

As referred to in the General Note in Lesson 1 of this Training Manual, there is often a choice when selecting Grade 1 indicators, with any of the options of Grade 1 mode (symbol, word or passage) being equally correct. However, for the purpose of the UEB Online exercises that are associated with this UEB Mathematics training manual, use the following criteria for implementing Grade 1 mode when completing the online exercises:

- Use the Grade 1 **symbol** indicator when there is only one symbol in the sequence requiring a Grade 1 indicator.
- Use the Grade 1 **word** indicator when there are two or more symbols in the sequence requiring Grade 1 mode except in a context whereby any literary elements will be affected and as a consequence will also be uncontracted.
- Use Grade 1 **passage** indicator (with Grade 1 terminator) when a comparison indicator or a space is used in the sequence and Grade 1 indicators are required on both sides of the equation. Remember however, the impact of Grade 1 mode on any literary elements.

Exercise 5

1. $\sqrt{25} = 5$
2. $\sqrt{3^2} + 4$
3. $3\sqrt{28} - 2\sqrt{7}$
4. $\sqrt{2t} \times \sqrt{8} = 4\sqrt{3}$
5. $\sqrt{16t} = 4\sqrt{3}$
6. $\frac{6}{2\sqrt{3}}$
7. $5(2\sqrt{3} + 3)$
8. $\sqrt{a^2 + b^2)}$
9. $E = \sqrt{1 - \frac{b^2}{a^2}}$
10. $V = \sqrt{2gR}$

Review Exercise 5

1. $x = \sqrt{x^2}$
2. $x = \frac{5 + \sqrt{45}}{2}$
3. $(a - 4)^2$
4. $(b - 4) = \pm\sqrt{3}$
5. $-(5x - 3) \geq 2$
6. $F = \frac{mv^2}{r}$
7. $m = \frac{y_2 - y_1}{x_2 - x_1}$
8. 10^{-4}
9. $(2r)^{\frac{1}{2}}$
10. $T_n = a + (n - 1)d$

Lesson 6: Shape Indicators (continued) and Miscellaneous Symbols

Shape Indicators

The shape indicators shown below also have a contracted (Grade 2) meaning and so have been shown with a Grade 1 indicator in front of them. A Grade 1 indicator is used to set Grade 1 mode when the Grade 1 meaning of the symbol could be misread as a contraction meaning and Grade 1 mode has not already been established in the sequence. For more information about Grade 1 mode, refer to Lesson 1 of this Training Manual and also Lesson 3 of *Unified English Braille Training Manual: Introductory Mathematics*.

⠰⠫ shape indicator

⠰⠻ shape termination indicator (there is no print representation for this braille symbol)

⠰⠫⠰⠫ □ square (as introduced in Lesson 6, *Unified English Braille Training Manual: Introductory Mathematics*)

⠰⠫⠰⠫ △ triangle, equilateral (as introduced in Lesson 6, *Unified English Braille Training Manual: Introductory Mathematics*)

⠰⠫⠰⠫⠄ ⬠ pentagon

⠰⠫⠰⠫⠆ ⬡ hexagon

⠰⠫⠰⠫⠖ ⯃ octagon

⠰⠫⠠⠰⠫ ▱ parallelogram

⠰⠫⠫ ○ circle (as introduced in Lesson 6, *Unified English Braille Training Manual: Introductory Mathematics*)

Transcriber defined shapes

⠠⠫ transcriber-assigned shape indicator

Note:

A transcriber-assigned shape indicator should not be used if the print symbol has already an assigned braille sign. The indicator should precede a short series of initials or a single Grade 1 word. The definitions of all the transcriber-assigned shape indicators used in the transcription should be available to the reader in either a transcriber's note or on a special page.

Example:

A smiling face icon could be shown in either of the following ways:

⠈⠫⠎⠋ or ⠈⠫⠎⠍⠊⠇⠑

Use of the shape termination indicator

⠰⠱ shape termination indicator

- If a shape is followed by a space, then no termination sign is required.
- If the shape is followed by punctuation or is unspaced from a following symbol, then the shape termination indicator must be used.
- The shape termination indicator has a Grade 2 (contracted) meaning so unless you are already in Grade 1 mode, a Grade 1 symbol indicator will be required before the shape termination indicator.
- Remember the numeric indicator (⠼) also sets Grade 1 mode for the remainder of the symbols-sequence.

Examples:

∆ ABC

⠰⠫⠼⠉ ⠠⠠⠁⠃⠉

∆ABC

⠰⠫⠼⠉⠱⠠⠠⠁⠃⠉

Miscellaneous Symbols (continued)

Not all miscellaneous symbols are included in this course. For a more extensive list of miscellaneous symbols and their equivalent braille representations, please refer to the “Unified English Braille Guidelines for Technical Material” (International Council on English Braille, 2014).

⠶	′	foot or minute (shown as a prime sign)
⠶⠶	″	inch or second (shown as a double prime sign)
⠖	!	factorial sign
⠰⠠⠡	∴	therefore sign
⠸⠪	∠	angle sign
⠼⠇	‖	is parallel to
⠼⠤	⊥	is perpendicular to

⠸⠿	≡	is congruent/equivalent to (three horizontal lines)
⠼⠸⠇	|||	is similar to (three vertical lines)
⠨⠣	[	open square bracket
⠨⠜	]	close square bracket
⠸⠣	{	open curly bracket
⠸⠜	}	close curly bracket
⠐		dot 5 continuation indicator (used when the braille sequence is too long for the line and needs to be broken)
⠬		visible blank space (an omission, which often occurs in fractions)
⠈⠱	/	cancelling sign
⠘⠲	.	recurring decimal (dot over the top of a number)
⠈⠩	✓	tick sign (not to be confused with the root sign)

Notes:

1. In general, the spacing of symbols follows the print.
2. Some of the signs listed above have a Grade 2 (contracted) meaning. A Grade 1 indicator will therefore be necessary if the sequence is not already in Grade 1 mode.
3. If the braille representation for a print sequence does not fit on the line, then a dot 5 continuation indicator placed at a logical place immediately following the last character may be required to show the braille is continuing for the remainder of the print sequence. Usually the preferred place to break is before a comparison sign or an operation sign.
4. Braille grouping signs are needed for the recurring decimal to explicitly show the character that has the dot above.
5. Remember when both a subscript and superscript are shown together in print, they are brailled with the subscript first followed by the superscript.

Examples:

$6'3''$

⠼⠋⠶⠼⠉⠶⠶

$\frac{2}{8} = \frac{\ }{4}$

⠼⠃⠌⠓ ⠐⠶ ⠰⠰⠷⠬⠨⠌⠼⠙⠾

OR

⠼⠃⠌⠓ ⠐⠶ ⠰⠷⠰⠬⠨⠌⠼⠙⠾

$4! \;=\; 4 \cdot 3 \cdot 2 \cdot 1$ (shown using a running product)

⠼⠙⠖ ⠐⠶ ⠼⠙⠐⠲⠼⠉⠐⠲⠼⠃⠐⠲⠼⠁

$\therefore\; x \;=\; 2$

⠰⠠⠡ ⠰⠭ ⠐⠶ ⠼⠃

$\angle A +\ \angle B = 90°$

⠸⠪⠠⠁⠐⠖⠸⠪⠠⠃ ⠐⠶ ⠼⠊⠚⠘⠚

$AB \parallel CD$

⠰⠠⠠⠁⠃ ⠼⠇ ⠰⠠⠠⠉⠙

$AB \perp CD$

⠰⠠⠠⠁⠃ ⠼⠤ ⠰⠠⠠⠉⠙

$\Delta ABC \;\equiv\; \Delta DEF$

⠰⠰⠰⠫⠼⠉⠱⠠⠠⠁⠃⠉ ⠸⠿ ⠫⠼⠉⠱⠠⠠⠙⠑⠋⠰⠄

OR

⠰⠫⠼⠉⠱⠠⠠⠁⠃⠉ ⠸⠿ ⠰⠫⠼⠉⠱⠠⠠⠙⠑⠋

$\Delta ABC \,|||\, \Delta DEF$

⠰⠰⠰⠫⠼⠉⠱⠠⠠⠁⠃⠉ ⠼⠸⠇ ⠫⠼⠉⠱⠠⠠⠙⠑⠋⠰⠄

OR

⠰⠫⠼⠉⠱⠠⠠⠁⠃⠉ ⠼⠸⠇ ⠰⠫⠼⠉⠱⠠⠠⠙⠑⠋

$$\frac{6 \times 3}{12} = \frac{\not{6}\ \times 3}{\not{6}\ \times 2}$$

⠰⠰⠰⠷⠼⠋⠐⠦⠼⠉⠨⠌⠼⠁⠃⠾

⠐⠶ ⠷⠼⠋⠈⠱⠐⠦⠼⠉⠨⠌⠼⠋⠈⠱⠐⠦⠼⠃⠾⠰⠄

OR

Note the use of the dot 5 continuation indicator in the following example which may not be necessary when the = sign is taking the whole of the expression to the next line.

OR

Yes ✓

In the following examples the recurring decimal has been shown using the braille grouping signs.

$0.\dot{7}$

$0.48\dot{3}4\dot{5}$

OR

In the same example as above, and because the digits are implicitly grouped in print with recurring decimals, this can also be shown as:

$0.48\dot{3}4\dot{5}$

Remember:

As referred to in the General Note in Lesson 1 of this Training Manual, there is often a choice when selecting Grade 1 indicators in mathematical contexts, with any of the options of Grade 1 mode (symbol, word or passage) being equally correct. However, for the purpose of the UEB Online exercises associated with this training manual, use the following criteria for implementing Grade 1 mode:

- Use the Grade 1 **symbol** indicator when there is only one symbol in the sequence requiring a Grade 1 indicator.
- Use the Grade 1 **word** indicator when there are two or more symbols in the sequence requiring Grade 1 mode except in a context whereby any literary elements will be affected and as a consequence will also be uncontracted.
- Use Grade 1 **passage** indicator (with Grade 1 terminator) when a comparison indicator or a space is used in the sequence and Grade 1 indicators are required on both sides of the equation. Remember however, the impact of Grade 1 mode on any literary elements.

Exercise 6

1. $0.35\dot{1}2\dot{3}$
2. $\angle EFG$ is adjacent to $\angle GFH$
3. $\triangle ABC \equiv \triangle DEF$
4. $\angle OCA \neq \angle OCB$
5. $3(8+5) = 3 \times \square + 3 \times \square$
6. $EF \parallel GH$
7. $\therefore PQ \perp RS$
8. {Craig, Frances, Tarna}
9. $\angle ABC = \angle DEF$
10. $\triangle EDF \,|||\, \triangle FDG$

Review Exercise 6

1. $(x+5)^2 = x^2 \square x + 25$
2. $P(x) = 5 - 3x + x^2$
3. $(9y+1)(7y+2) = 0$
4. $v = \sqrt{\frac{g}{k}}$
5. $A = \frac{1}{2}h(a+b)$
6. $S = \frac{n}{2}(a+l)$
7. $\frac{2y}{5} = 11 - \frac{y}{3}$
8. $T_n = ar^{n-1}$
9. $\sqrt{8.41 \times 10^{-8}}$
10. $(xy)^3$

Lesson 7: Functions

Trigonometric functions

Common trigonometric functions are Sine, Cosine and Tangent. These functions are usually abbreviated in print as sin, cos and tan.

1. Sine (Sin), Cosine and Tangent may be contracted unless already in Grade 1 mode.
2. Where the function name is **preceded** or **followed** by a lowercase letter, a space may be needed between the letter and the function to remove any ambiguity as to where the function name begins and ends.

Examples:

⠎⠊⠝ ⠭ $\sin x$

⠉⠕⠎ ⠽ $\cos y$

⠭ ⠎⠊⠝ ⠽ $x\sin y$

If the function name is directly preceded or followed by a number, then the number should be written **unspaced** from the function name.

Remember the numeric indicator (⠼) also sets Grade 1 mode for the next symbols-sequence.

Examples:

⠼⠑⠞⠁⠝⠲⠙⠑⠛ $5\tan 45°$

⠼⠛⠎⠊⠝⠲⠙⠑⠛ $7\sin 45°$

⠼⠎⠊⠝⠲⠙⠑ $\sin 45$

⠭ ⠞⠁⠝⠲⠁⠚ $x\tan 60$

⠭ ⠎⠊⠝⠲⠁⠚ $x\sin 60$

A space is not needed if the function name is already separated by a bracket or by a braille indicator, such as a capitalisation indicator, a fraction indicator, or a Greek letter indicator, which is explained more fully in Lesson 8 of this Training Manual.

Remember the opening and closing fraction indicators have a Grade 2 (contracted) meaning so unless you are already in Grade 1 mode, a Grade 1 symbol indicator will be required before the opening and closing fraction indicator.

Examples:

⠎⠔⠐⠣⠠⠁⠐⠖⠠⠃⠐⠜ $\sin(A + B)$

⠠⠉⠕⠎⠠⠁ $\text{Cos}A$

⠠⠞⠁⠝⠼⠑⠚⠘⠚ $\text{Tan}50°$

⠇⠕⠛⠰⠷⠭⠨⠌⠼⠃⠾ $\log \frac{x}{2}$

⠎⠔⠨⠹ $\sin\theta$

Logarithmic functions

The logarithmic function is usually written as log or Log and may be followed by a subscript indicating the base. A logarithm to base “e” is called a natural log and is often abbreviated to ln.

Examples:

$5\log x$

⠼⠑⠇⠕⠛ ⠰⠭

$\log_2 8 = 3$

⠇⠕⠛⠰⠢⠼⠃⠼⠓ ⠐⠶ ⠼⠉

$\ln e = 1$

⠇⠝ ⠰⠑ ⠐⠶ ⠼⠁

Remember:

As referred to in the General Note in Lesson 1 of this Training Manual, there is often a choice of Grade 1 indicators in mathematical contexts, with any of the options of Grade 1 mode (symbol, word or passage) being equally correct. However, for the purpose of the UEB Online exercises that are associated with this training manual, use the following criteria for implementing Grade 1 mode:

- Use the Grade 1 **symbol** indicator when there is only one symbol in the sequence requiring a Grade 1 indicator.
- Use the Grade 1 **word** indicator when there are two or more symbols in the sequence requiring Grade 1 mode except in a context whereby any literary elements will be affected and as a consequence will also be uncontracted.
- Use Grade 1 **passage** indicator (with Grade 1 terminator) when a comparison indicator or a space is used in the sequence and Grade 1 indicators are required on both sides of the equation. Remember however, the impact of Grade 1 mode on any literary elements.

Exercise 7

1. $\text{Sin}\, A$
2. $\text{Log}(x + y)$
3. $6\tan 90°$
4. $\text{Sin}30$
5. $5\text{Sin}45$
6. $7\cos 5x$
7. $\log a + \log b = \log ab$
8. $\log_4 = -2$
9. $\frac{\sin C}{c} = \frac{\sin A}{a}$
10. $\frac{\sin Z}{35} = \frac{\sin 70°}{7}$

Review Exercise 7

1. $\frac{5}{\square} = \frac{30}{36}$
2. $\text{In } \triangle DEF, \angle E = 90°$
3. $\sqrt{9a^8}$
4. $(\frac{25}{49})^{\frac{1}{2}}$
5. $\sqrt{m^2}$
6. $\frac{18\cos 12°}{13\tan 68°}$
7. $\sqrt[6]{101.9}$
8. $\text{In } \triangle KLM, \angle M = 27°51'$
9. $\sin Z = \frac{35\sin 70°}{45}$
10. $\log_e(\frac{2}{3})$

Lesson 8: Greek Letters

Greek letters are used extensively in Mathematics. While only a small number have been used in this Training Manual, the principles for use remain the same for all. Refer to the “Unified English Braille Guidelines for Technical Material” (International Council on English Braille, 2014) for a complete list.

⠨⠁	α	alpha (lower case)	⠨⠨⠁	A	Capital
⠨⠃	β	beta (lower case)	⠨⠨⠃	B	Capital
⠨⠙	δ	delta (lower case)	⠨⠨⠙	Δ	Capital
⠨⠑	ε	epsilon (lower case)	⠨⠨⠑	E	Capital
⠨⠛	γ	gamma (lower case)	⠨⠨⠛	Γ	Capital
⠨⠹	θ	theta (lower case)	⠨⠨⠹	Θ	Capital
⠨⠇	λ	lambda (lower case)	⠨⠨⠇	Λ	Capital
⠨⠍	μ	mu (lower case)	⠨⠨⠍	M	Capital
⠨⠏	π	pi (lower case)	⠨⠨⠏	Π	Capital
⠨⠎	σ	sigma (lower case)	⠨⠨⠎	Σ	Capital

Remember:

As referred to in the General Note in Lesson 1, there is often a choice of Grade 1 indicator, with any of the options of Grade 1 mode (symbol, word or passage) being equally correct. However, for the purpose of the UEB Online exercises associated with this training manual, use the following criteria for implementing Grade 1 mode:

- Use the Grade 1 **symbol** indicator when there is only one symbol in the sequence requiring a Grade 1 indicator.
- Use the Grade 1 **word** indicator when there are two or more symbols in the sequence requiring Grade 1 mode except in a context whereby any literary elements will be affected and as a consequence will also be uncontracted.
- Use Grade 1 **passage** indicator (with Grade 1 terminator) when a comparison indicator or a space is used in the sequence and Grade 1 indicators are required on both sides of the equation. Remember however, the impact of Grade 1 mode on any literary elements.

Exercise 8

1. $A = \frac{\theta}{360} \times 2\pi r$
2. $V = \frac{1}{3}\ \pi r^2\ h$
3. $A = 2\pi r^2 + 2\pi rh$
4. $S = \ \theta r(r + l)$
5. $\therefore\ \Sigma x = 195$
6. $V = \frac{1}{3}\ \pi$
7. $C = 2\pi r$
8. Standard deviation (σ_n)
9. Mean $= \frac{\Sigma fx}{\Sigma f}$
10. surface area $= \pi rs +\ \pi r^2$

Review Exercise 8

1. $0\ \leq\ x\ \leq\ 2\pi$
2. $\frac{25\pi}{8} m^2$
3. $(\log_e x)^4$
4. $sin\theta\ =\ \frac{14.6\ \sin 48°}{12.6}$
5. $\angle DAC =\ \angle ACB$
6. $\therefore \triangle\ ABC \equiv \triangle\ ABD$
7. $\sqrt[3]{8u^{18}}$
8. $25gh\ \div\ \square = 5g$
9. $y = -\frac{1}{2}(x + 4)(2 - x)$
10. $DE \parallel AC$ and $CE{:}EB$ = 2:3

Lesson 9: Review Test

Congratulations on reaching this Review lesson. The content of each lesson has built upon preceding lessons, with the overall structure of this Training Manual designed to reinforce several foundational principles, including the following:

1. The numeric indicator (⠼) sets numeric mode **and** Grade 1 mode for the remainder of the symbols-sequence.
2. Numeric mode is transitive over the 10 digits, the full stop, the comma, the numeric space, the simple fraction line and the continuation indicator and is terminated by symbols such as the hyphen, dash, slash/oblique stroke and colon whereby the numeric indicator will need to be repeated.
3. When Grade 1 mode is established by the numeric indicator, it is terminated by a space, hyphen, dash and Grade 1 terminator.
4. A braille symbol may have both an uncontracted (Grade 1) meaning and a contracted (Grade 2) meaning. A Grade 1 indicator is used to set Grade 1 mode when the Grade 1 meaning of a symbol could be misread as a contraction meaning or a numeric meaning.
5. Understanding of when a fraction is considered a “simple fraction”. If the sequence does not satisfy the definition of a simple fraction, then it must be treated as a general fraction.
6. The rules associated with “the next item” should be considered and are particularly relevant in the use of subscripts and superscripts.

Remember:

As referred to in the General Note in Lesson 1 of the UEB Training Manual: Advanced Mathematics and throughout the preceding lessons, the choice of options for Grade 1 mode (symbol, word or passage) are equally correct in mathematical contexts. However, for the purpose of completing the following Review Test in the UEB Online course, use the following criteria for implementing Grade 1 mode:

- Use the Grade 1 **symbol** indicator when there is only one symbol in the sequence requiring a Grade 1 indicator.
- Use the Grade 1 **word** indicator when there are two or more symbols in the sequence requiring Grade 1 mode except in a context whereby any literary elements will be affected and as a consequence will also be uncontracted.
- Use Grade 1 **passage** indicator (with Grade 1 terminator) when a comparison indicator or a space is used in the sequence and Grade 1 indicators are required on both sides of the equation. Remember however, the impact of Grade 1 mode on any literary elements.

Review Test

1. $A = \pi r^2$
2. $\frac{4\sqrt{3}\times\sqrt{18}}{\sqrt{12}}$
3. $time = \frac{distance}{speed}$
4. $4r^4 \div 2r$
5. $x^{7.9}$
6. $A = P(1+r)^n$
7. $3\sqrt{8}$
8. x^{-2}
9. x^3y
10. $(2x+9)^2$
11. $\frac{x^{\frac{1}{4}}}{4}$
12. $\sqrt[3]{2x}$
13. $\{2, 4, -6\}$
14. $P(E) = \frac{n(E)}{n(S)}$
15. $\sqrt{96} \div \sqrt{12}$
16. $y = -\frac{1}{2}(x-1)^5$
17. $AB \parallel PQ$
18. $\therefore \triangle MNO \equiv \triangle PQR$
19. $3! = 3 \cdot 2 \cdot 1$
20. $\frac{3x^2-6x}{x^2+x-6}$

Lesson 10: Advanced Test

Congratulations on reaching this Advanced Test. The content of each lesson has built upon preceding lessons, with the overall structure of this Training Manual designed to reinforce several foundational principles, including the following:

1. The numeric indicator (⠼) sets numeric mode **and** Grade 1 mode for the remainder of the symbols-sequence.
2. Numeric mode is transitive over the 10 digits, the full stop, the comma, the numeric space, the simple fraction line and the continuation indicator and is terminated by symbols such as the hyphen, dash, slash/oblique stroke and colon whereby the numeric indicator will need to be repeated.
3. When Grade 1 mode is established by the numeric indicator, it is terminated by a space, hyphen, dash and Grade 1 terminator.
4. A braille symbol may have both an uncontracted (Grade 1) meaning and a contracted (Grade 2) meaning. A Grade 1 indicator is used to set Grade 1 mode when the Grade 1 meaning of a symbol could be misread as a contraction meaning or a numeric meaning.
5. Understanding of when a fraction is considered a “simple fraction”. If the sequence does not satisfy the definition of a simple fraction, then it must be treated as a general fraction.
6. The rules associated with “the next item” should be considered, and are particularly relevant in the use of subscripts and superscripts.

Remember:

As referred to in the General Note in Lesson 1 of the UEB Training Manual: Advanced Mathematics, and throughout the preceding lessons, the choice of options for Grade 1 mode (symbol, word or passage) are equally correct in mathematical contexts. However, for the purpose of completing the following Advanced Test in the UEB Online course, use the following criteria for implementing Grade 1 mode:

- Use the Grade 1 **symbol** indicator when there is only one symbol in the sequence requiring a Grade 1 indicator.
- Use the Grade 1 **word** indicator when there are two or more symbols in the sequence requiring Grade 1 mode except in a context whereby any literary elements will be affected and as a consequence will also be uncontracted.
- Use Grade 1 **passage** indicator (with Grade 1 terminator) when a comparison indicator or a space is used in the sequence and Grade 1 indicators are required on both sides of the equation. Remember however, the impact of Grade 1 mode on any literary elements.

Advanced Test

1. $\sqrt{a^2 + b^2}$
2. $\sqrt[3.4]{8}$
3. The mean $= \frac{\Sigma fx}{\Sigma f}$
4. $y = x^4 - x^3 - 10x^2 - 8x$
5. $\log \frac{1+x}{x}$
6. $\frac{a}{\sin 20°} = \frac{12}{\sin 60°}$
7. $y = log_a f(x)$
8. $f(L) = 2\pi\sqrt{\frac{L}{g}}$
9. $f(x) = 2\, cos\, \left(x + \frac{\pi}{2}\right) + 1$
10. $\log(x^2 - x - 2) - \log(x + 1)$
11. $\log\sqrt{x^2 - 4x + 4}$
12. $\log_a \frac{x}{\sqrt{y}}$
13. $\sin 120° = 0.87$
14. $\tan\, \theta = \frac{\sin\, \theta}{\cos\, \theta}$
15. $\cos\, \frac{\pi}{5} = \frac{1+\sqrt{5}}{4}$
16. $\log_e x$
17. $m = \frac{y_2 - y_1}{x_2 - x_1}$
18. $\frac{1}{k(k+1)(k+2)}$
19. $\frac{4.1 \times \sqrt{48.12}}{26.23}$
20. $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

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