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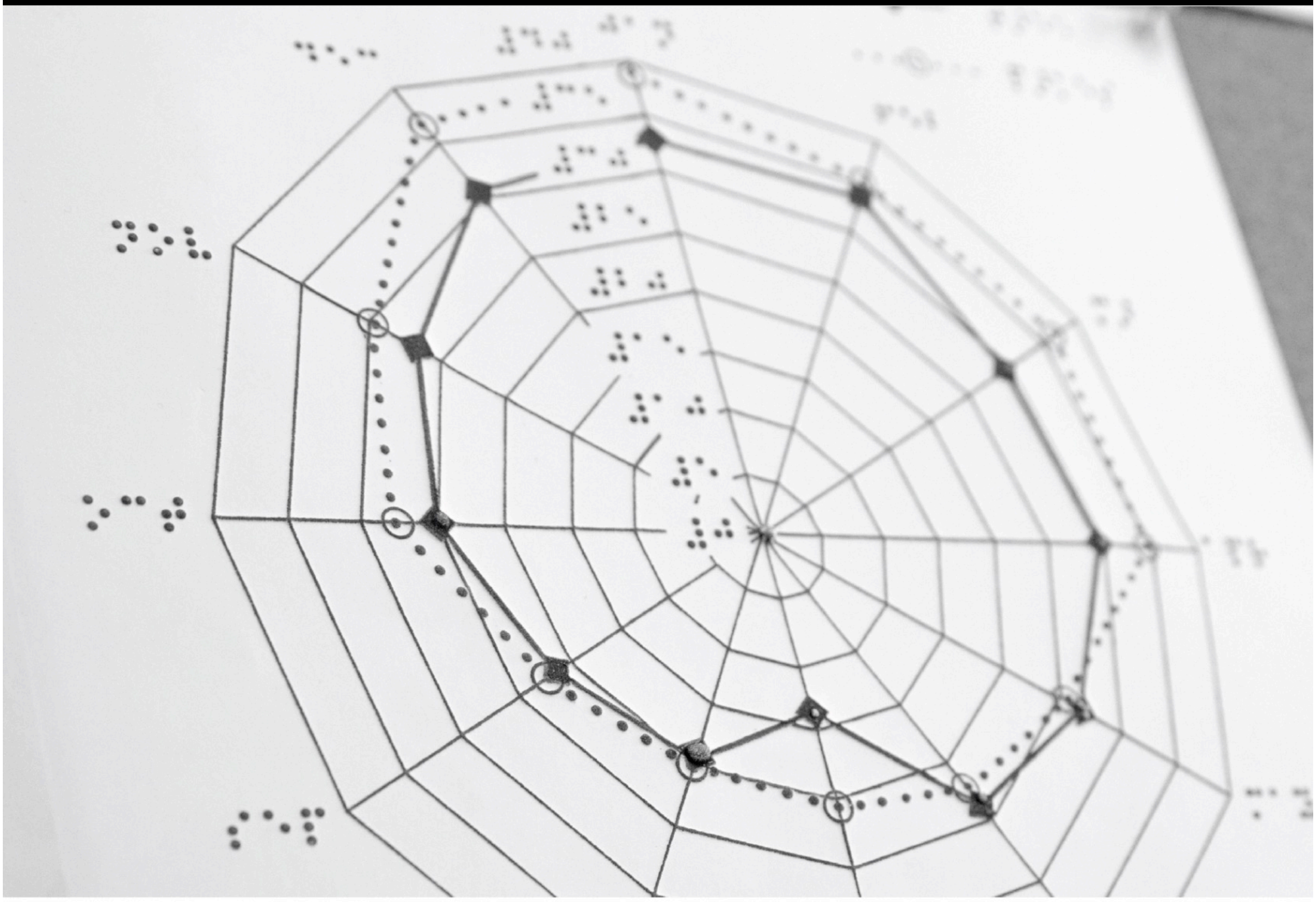
Advancing the RIDBC Renwick Centre experience.

Unified English Braille Training Manual

Extension Mathematics

Josie Howse



UEBOnline

Unified English Braille Training Manual: Extension Mathematics
Revision 7

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NextSense

361-365 North Rocks Road

North Rocks NSW 2151

Australia

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NextSense Institute is Australia’s leading centre for research and professional studies in the field of education for children with sensory disabilities; offering webinars, short courses, and degree programs for parents, carers, educators, and health professionals. NextSense Institute is committed to providing high-quality teaching and learning opportunities. Our programs are conducted by leading national and international experts for education and health professionals who support people who are deaf, hard of hearing, blind or have low vision.

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Forewordh

We live in an era where education leaders and decision makers are responding to national and international disability anti-discrimination legislation, policies and standards that prioritise equitable inclusive education for all, leaving no-one behind. However, for learners with vision impairment, the low incidence nature of their disabilities can present challenges for education systems and schools. The World Blind Union in 2019 highlighted three global challenges impeding education enrolment and achievement for children and young people with vision impairment, in particular in low and middle-income countries. These challenges are a chronic shortage of teachers who are qualified to teach braille, an absence of braille instruction in teacher training programs or offered by education systems, and the high cost of braille equipment. With these challenges in mind, NextSense has created online, open access training programs and resources in braille literacy and mathematics for education leaders, teachers, parents and caregivers – basically, for anyone who wants to learn braille.

In my experience as a lecturer in sensory impairment, professional qualifications, quality teaching and positive student outcomes are closely linked. Knowledgeable and inspiring teachers reach out to their students, building and enriching student knowledge and igniting curiosity, self-confidence, and motivation to pursue learning. In the field of mathematics, we need such teachers who can reach out to their students with vision impairment. Teacher training institutions have a responsibility to provide teachers with instruction in braille codes and braille literacy development, the mechanics of producing braille and tactile representations of visual information, and practical experience in reading and transcribing print and braille materials.

In offering this UEB Extension Mathematics training manual and online program, NextSense **aims** to address the following:

- To extend professional and parent/caregiver knowledge of braille mathematics by building upon the lessons and exercises presented in the introductory and advanced braille mathematics training programs
- To introduce mathematical symbols in braille that are commonly found in senior secondary mathematics publications; and in so doing,
- To support and promote equitable access to mathematics for learners who use braille or a combination of braille, print and digital formats.

In Australia, as in other parts of the world, braille experts who are knowledgeable in the field of mathematics are limited in number. NextSense is fortunate to have Josie Howse write the content of this Extension Mathematics training manual, in addition to the Introductory and Advanced Mathematics training manuals. During her employment and professional affiliations over the past four decades, Josie Howse has been at the forefront of braille training, assessment, and production in Australia and overseas. She has contributed to the development of Unified English Braille (UEB) by the International

Council on English Braille (ICEB), and adoption and implementation of Unified English Braille by Australia in 2005.

NextSense acknowledges the contribution of Craig Cashmore of PeppaCode in developing and maintaining the UEB Online website and translating the content of the literary and mathematics training manuals into accessible, interactive online training programs. Sincere thanks are also extended to the NextSense UEB Online design team and the foundations and trusts that have contributed to the UEB Online training programs since 2014.

I wish to reflect on a quotation from Hellen Keller, as published by the American Foundation for the Blind. Hellen Keller spoke these words 14 years before the United Nations “Universal Declaration of Human Rights” in 1948. Her words are just as relevant today and serve as a clarion call to us all to continue our efforts to promote and support the right to education for all children, leaving no-one behind.

> "Education should train the child to use his brains, to make for himself a place in the world and maintain his rights even when it seems that society would shove him into the scrap-heap."
>
> Hellen Keller, 1934

Frances Gentle, AO, PhD, D. Litt. *Honoris Causa*
Conjoint Lecturer, NextSense Institute,
President, International Council for Education of People with Visual Impairment; and Co-President, South Pacific Educators in Vision Impairment

Contributors

Author: Josie Howse, PSM

M.Spec. Ed. (Sensory Disability) – University of Newcastle,
BA (Ed.) – Macquarie University

Josie Howse is an Adjunct Research Fellow with the NextSense Institute and former Manager of the Braille and Large Print Services, NSW Department of Education. The NSW Department of Education team provides all texts and examinations in braille, large print and e-text to students with vision impairment in the government sector and is the largest producer of alternate format student textbooks and examinations in Australia.

Josie has been working in the field of vision impairment for more than 40 years. She has held a number of executive positions at national and state levels and has extensive experience in braille code development at an international level. Josie was the editor of the 2006 Unified English Braille Primer: Australian Edition, and co-editor of the 2016 Unified English Braille: Australian Training Manual.

Josie was awarded the Public Service Medal (PSM) in the 2007 Queen's Birthday Honours list, has been listed annually in the Who's Who of Australian Women since 2008, and is the 2012 recipient of a Lifetime Achievement Award from the Round Table on Information Access for People with Print Disabilities, and awarded Honorary Life Membership of the South Pacific Educators in Vision Impaired (SPEVI) in 2020.

UEB Online Developer: Craig Cashmore

B. Eng. (Hons) - UTS

Craig holds an Engineering Degree in Telecommunications and has worked in the software development industry for over 30 years, holding senior software design, software architecture and technical management positions in companies including Jtec, Ericsson and LongReach Networks.

More recently Craig founded PeppaCode, a web and app development business focused on 'out-of-the-ordinary' strategic web and software development for small business, start-ups and educational institutions.

Some of Craig's achievements at PeppaCode include the successful launch of UEB Online for NextSense and a vehicle tracking and management system for a bus operator. Craig continues to work on new and innovative projects using modern web technologies.

Lesson 1 - Revision

Welcome to the Unified English Braille (UEB) Training Manual: Extension Mathematics. This training program builds on the basic principles that you have already learnt in the Introductory and Advanced level training programs but will apply the principles in an extended mathematics context. You will also be introduced to some new signs which may be required when transcribing higher level mathematics.

All examples and test items presented in this training program have been taken directly from Senior Mathematics textbooks.

Lesson 1 will review some of these critical principles to reinforce what has already been delivered in the Introductory and Advanced mathematics programs and prepare you for implementing best practice in this extension training program.

Some of the topics covered in this revision lesson are as follows:

- Grade 1 Mode
- Shape Indicators
- Fractions
- Miscellaneous Symbols
- Superscripts and Subscripts

Grade 1 Mode

The rules for the use of Grade 1 mode in a literary context will also apply in a mathematical context and are summarised below.

A braille symbol may have both an uncontracted (Grade 1) meaning and a contracted (Grade 2) meaning. A Grade 1 indicator is used to set Grade 1 mode when the Grade 1 meaning of a symbol could be misread as a contraction meaning or a numeric meaning.

The extent of Grade 1 mode is determined by the Grade 1 indicator in use.

⠰	Grade 1 Symbol Indicator (dots 5 6)
⠰⠰	Grade 1 Word Indicator (dots 5 6, 5 6)
⠰⠰⠰	Grade 1 Passage Indicator (dots 5 6, 5 6, 5 6)
⠰⠄	Grade 1 Passage Terminator (dots 5 6, 3)
⠐⠐⠿⠰⠰⠰	Grade 1 Passage Indicator on a line of its own
⠐⠐⠿⠰⠄	Grade 1 Passage Terminator on a line of its own

Note:

- Grade 1 indicators will not be needed for simple arithmetic problems involving numbers, operation signs, numerical fractions, and mixed numbers.

Grade 1 Symbol Indicator

- A Grade 1 symbol indicator (⠰) sets Grade 1 mode for only the next symbol.

  Example:

  In Jones^{15} you will find the reference.

  ⠠⠔ ⠠⠚⠐⠕⠎⠰⠔⠼⠁⠑ ⠽ ⠺ ⠋⠔⠙ ⠮
  ⠗⠑⠋⠻⠰⠑⠲

- A Grade 1 symbol indicator is NOT required before the letters a, i and o, because they do not have a contraction meaning when they "stand alone".

  Example:

  The letters a, i and o do not need a Grade 1 indicator.

  ⠠⠮ ⠇⠗⠎ ⠁⠂ ⠊ ⠯ ⠕ ⠙ ⠝ ⠝⠑⠫ ⠁
  ⠠⠛⠗⠁⠙⠑ ⠼⠁ ⠔⠙⠊⠉⠁⠞⠕⠗⠲

- A Grade 1 symbol indicator precedes a capitalisation indicator.

  Example:

  The letter B.

  ⠠⠮ ⠇⠗ ⠰⠠⠃⠲

- A Grade 1 indicator will be required when a lower-case letter a – j immediately follows a digit so it will not be misread as part of the number.

  Examples:

  25b ⠼⠃⠑⠰⠃ not 252

  Trevor@take5.com

  ⠠⠞⠗⠑⠧⠕⠗⠈⠁⠞⠁⠅⠑⠼⠑⠲⠰⠉⠕⠍ not 5.3

- A Grade 1 symbol indicator may be required for a sequence of letters in braille that could represent an alphabetic wordsign, or a shortform such as *ab* (about) or *ac* (according), if the letter or letter sequence is "standing alone".

  Example:

ab = ac ⠰⠁⠃ ⠐⠶ ⠰⠁⠉

- Remember that the numeric indicator, besides setting numeric mode, also sets Grade 1 mode for the next symbols-sequence.

  Example:

  www.shopping4you

  ⠺⠺⠺⠲⠩⠕⠏⠏⠬⠼⠙⠽⠕⠥

Termination of Grade 1 Mode

- When Grade 1 mode is set by a numeric indicator, it is terminated by a space, a hyphen, a dash, or a Grade 1 terminator.

  Examples:

  3-dimensional

  ⠼⠉⠤⠙⠊⠍⠢⠨⠝⠁⠇

  3–D

  ⠼⠉⠠⠤⠰⠠⠙

  www.education4everyone/parents-students

  ⠺⠺⠺⠲⠫⠥⠉⠁⠰⠝⠼⠙⠰⠑⠧⠑⠗⠽⠕⠝⠑⠸⠌⠐

  ⠏⠁⠗⠑⠝⠞⠎⠤⠌⠥⠙⠢⠞⠎

Numeric Mode

- Remember that the following symbols may occur in numeric mode:
  - ➢ The ten digits
  - ➢ Full stop (period)
  - ➢ Comma
  - ➢ The numeric space
  - ➢ Simple numeric fraction line, and
  - ➢ The continuation indicator

Termination of Numeric mode

- Therefore, the numeric mode is terminated by a space or any symbol such as the hyphen, dash, slash/oblique stroke, and colon.

1915-19

⠼⠁⠊⠁⠑⠤⠼⠁⠊

1915–1919

⠼⠁⠊⠁⠑⠠⠤⠼⠁⠊⠁⠊

20/20

⠼⠃⠚⠸⠌⠼⠃⠚

16:30 pm

⠼⠁⠋⠒⠼⠉⠚ ⠏⠍

Standing Alone Rule

Refer to *The Rules of Unified English Braille* (Simpson, 2013) for a comprehensive definition and explanation of usage of “standing alone”.

A letter or letters-sequence is considered to be “standing alone” when:

- it is **preceded** and **followed** by a space, a hyphen, or a dash (of any length); or
- common punctuation and indicator symbols intervene between the letter or letter-sequence and the **preceding** or **following** space, hyphen, or dash; or
- a word with an interior apostrophe is considered to be “standing alone” under the specific provisions stated in *The Rules for Unified English Braille*, 2013 referring to contractions, alphabetic wordsigns, strong wordsigns and shortforms.

Examples:

b-3

⠰⠃⠤⠼⠉

B–3

⠰⠠⠃⠠⠤⠼⠉

“b”

⠦⠰⠃⠴

(b)

⠐⠣⠰⠃⠐⠜

(B)

⠐⠣⠰⠠⠃⠐⠜

“a’s and b’s”

⠦⠁⠄⠎ ⠯ ⠰⠃⠄⠎⠴

“A’s and B’s”

⠦⠠⠁⠄⠎ ⠯ ⠰⠠⠃⠄⠎⠴

“can’t”

⠦⠉⠄⠞⠴

Grade 1 Word Indicator

- The Grade 1 word indicator (⠰⠰) sets Grade 1 mode for the next symbols-sequence or the remainder of the current symbols-sequence.
- The effect of a Grade 1 word indicator is terminated by a space or a Grade 1 terminator.
- Remember that the numeric indicator also sets Grade 1 mode for the next symbols-sequence.
- If a complex algebraic expression does NOT include a comparison sign (such as an equal's sign) then it is unlikely to include interior spaces in braille. In such cases, a Grade 1 word indicator will be enough to ensure that superscripts, subscripts, fractions, radicals, arrows, and shape indicators are well defined without the need for Grade 1 symbol indicators.

  Examples:

- Evaluate $\sqrt{(y\ -x^2)}$

  ⠠⠑⠧⠁⠇⠥⠁⠞⠑ ⠰⠰⠩⠐⠣⠽⠐⠤⠭⠔⠼⠃⠐⠜⠬

- Replace I with $\frac{E}{R}$

  ⠠⠗⠑⠏⠇⠁⠉⠑ ⠠⠊ ⠾ ⠰⠰⠷⠠⠑⠨⠌⠠⠗⠾

Grade 1 Passage Indicator

- A Grade 1 passage indicator (⠰⠰⠰) sets Grade 1 mode for the next passage.

- Complex algebraic equations that include a comparison sign are best enclosed in Grade 1 passage indicators. This will ensure that isolated letters and indicators such as superscripts, subscripts, fractions, radicals, arrows, and shapes are well defined without the need for Grade 1 symbol indicators.
- A Grade 1 passage is terminated by the Grade 1 passage terminator **(** ⠰⠄)

Grade 1 Passage Terminator

- The Grade 1 passage terminator (⠰⠄) follows **immediately** after the last affected symbols-sequence of a Grade 1 passage.

  Example:

  $x = y - z, y = x + z, z = y - x$

  ⠰⠰⠰⠭ ⠐⠶ ⠽⠐⠤⠵⠂ ⠽ ⠐⠶ ⠭⠐⠖⠵⠂ ⠵ ⠐⠶ ⠽⠐⠤⠭⠰⠄

Grade 1 Passage Indicator and Terminator on lines of their own

- Where it is better to preserve the natural alignment of a text, for example in a computer program or a set of equations in mathematics, place the Grade 1 passage indicator (⠰⠰⠰) on a separate line above the equations (consider the margin as a suitable location), and the Grade 1 terminator (⠰⠄) on a separate line below the equations.
- Refer to the example below which shows this use with a set of quadratic equations using superscripts. When this method is used, each indicator should be preceded and followed by the dot locator for "use" ⠐⠐⠿.

  Example:

  Solve the following quadratic equations:

  1. $x^2 - x - 2 = 0$
  2. $x^2 - 4x - 3 = 0$
  3. $2x^2 - x = 1$

  ⠠⠎⠕⠇⠧⠑ ⠮ ⠋⠕⠇⠇⠪⠬ ⠟⠥⠁⠙⠗⠁⠞⠊⠉
  ⠑⠟⠥⠁⠰⠝⠎⠒

  ⠐⠐⠿⠰⠰⠰

⠼⠁⠲ ⠭⠔⠼⠃⠐⠤⠭⠐⠤⠼⠃ ⠐⠶ ⠼⠚

⠼⠃⠲ ⠭⠔⠼⠃⠐⠤⠼⠙⠭⠐⠤⠼⠉ ⠐⠶ ⠼⠚

⠼⠉⠲ ⠼⠃⠭⠔⠼⠃⠐⠤⠭ ⠐⠶ ⠼⠁

⠐⠐⠿⠰⠄

Shape Indicators

A number of shapes used in mathematics were introduced in the UEB Introductory and Advanced Mathematics training manuals. The most common shapes used in senior mathematics are shown below.

Please note that the shape indicator also has a contracted (Grade 2) meaning and so may require a Grade 1 indicator in front of it. A Grade 1 indicator is used to set Grade 1 mode when the Grade 1 meaning of the symbol could be misread as a contraction meaning and Grade 1 mode has not already been established in the sequence.

⠫		shape indicator
⠱		shape termination indicator (there is no print representation for this braille symbol)
⠫⠼⠙	□	square
⠫⠼⠉	△	triangle (equilateral)
⠫⠼⠑	⬠	pentagon
⠫⠼⠋	⬡	hexagon
⠫⠼⠓	⯃	octagon
⠫⠈⠼⠙	▱	parallelogram
⠫⠿	○	circle
⠬		visible space

Transcriber defined shapes

⠈⠫	transcriber-assigned shape indicator

Note: A transcriber-assigned shape indicator should **not** be used if the print symbol has already an assigned braille sign. The indicator should precede a short series of initials or a single Grade 1 word. The definitions of all the transcriber-assigned shape indicators used in the transcription should be available to the reader in either a transcriber's note or on a specific page.

Example:

A smiling face icon could be shown either of the following ways:

⠈⠫⠎⠋ or ⠈⠫⠎⠍⠊⠇⠑

Use of the shape termination indicator

⠱ Shape termination indicator (there is no print representation for this symbol in braille)

- If a shape indicator is followed by a space, then no termination sign is required.
- If the shape indicator is followed by punctuation or is unspaced from a following symbol, then the shape termination indicator must be used.
- All the initial shape indicators initiate shape mode so no further Grade 1 indicators will be needed.

Examples:

Δ ABC

⠰⠫⠼⠉ ⠠⠠⠁⠃⠉

Δ,

⠰⠫⠼⠉⠱⠂

ΔABC

⠰⠫⠼⠉⠱⠠⠠⠁⠃⠉

Omissions in technical expressions

Follow the print when omissions are indicated by shapes (such as squares) or punctuation (such as underscores or question marks).

When print uses a blank within a technical expression, use the visible space indicator (⠬) to represent the blank.

Space the sign of omission the same as the sign being omitted.

Remember a Grade 1 indicator is used to set Grade 1 mode when the Grade 1 meaning of the symbol could be misread as a contraction meaning and Grade 1 mode has not already been established in the sequence.

Examples:

□ + 75 = 80

⠰⠫⠼⠙⠱⠐⠖⠼⠛⠑ ⠐⠶ ⠼⠓⠚

48 = 12 × □ + Δ

⠼⠙⠓ ⠐⠶ ⠼⠁⠃⠐⠦⠫⠼⠙⠱⠐⠖⠫⠼⠉

? ÷ 6 = 84

⠰⠦⠐⠌⠼⠋ ⠐⠶ ⠼⠓⠙

50 − = 40

⠼⠑⠚⠐⠤⠬ ⠐⠶ ⠼⠙⠚

Fractions

It is important to understand the definition of a **simple fraction**, which the elements and only those elements that can be considered to be a simple fraction and subsequently use the simple fraction line, (⠌) between the numerator and the denominator.

If a fraction does **not** comply with the **simple fraction** definition (restated below) for whatever reason, then it will be a **general fraction** and will require a quite different approach, using different signs as described in a section that follows.

Simple numeric fraction

A simple numeric fraction is one whose numerator (top of the fraction line) and denominator (bottom of the fraction line) contain only:

- digits,
- decimal points,
- commas or
- separator spaces,
- **and**, if the fraction line in print (often referred to as the vinculum) is drawn between the two vertically (or near vertically) arranged numbers as shown in the print.

If the fraction complies fully with the definition above for a simple fraction, then a numeric fraction line symbol (⠌) should be used between the numerator and the denominator and continues the numeric mode.

Simple numeric fraction line symbol ⠌

The numeric indicator for a simple fraction, will **not** need to be repeated after the fraction line as numeric mode has been continued.

Examples:

⠼⠁⠌⠃	$\frac{1}{2}$	(vertically) or
⠼⠁⠌⠃	${}^{1}/_{2}$	(near vertically)
⠼⠁⠊⠋⠌⠃⠓	$\frac{196}{28}$	(vertically) or
⠼⠁⠊⠋⠌⠃⠓	${}^{196}/_{28}$	(near vertically)
⠼⠃⠲⠑⠚⠚⠌⠁⠚⠲⠚⠚⠚	$\frac{2.500}{10.000}$	(decimals)
⠼⠁⠚⠂⠚⠚⠚⠌⠑⠚⠂⠚⠚⠚	$\frac{10,000}{50,000}$	(commas)
⠼⠁⠚⠐⠚⠚⠚⠌⠑⠚⠐⠚⠚⠚	$\frac{10\ 000}{50\ 000}$	(separator spaces)

General Fractions

The braille symbols shown below for the opening and closing general fraction indicators have a contracted (Grade 2) meaning.

A Grade 1 indicator is used to set Grade 1 mode when the Grade 1 meaning of the symbol could be misread as a contraction meaning. The extent of Grade 1 mode is determined by the Grade 1 indicator in use.

General fraction indicators

⠷	Opening general fraction indicator
⠾	Closing general fraction indicator
⠨⠌	General fraction line

- If the numerator or denominator of a fraction is not entirely numeric, as defined above for a simple fraction, then the general fraction indicators should be used. Write the opening general fraction indicator (⠷), then the numerator (top) expression, then the general fraction line symbol (⠨⠌), then the denominator (bottom) expression and finally the closing general fraction indicator (⠾). See examples below.
- The numerator and denominator may be any kind of expression, including fractions of either simple numeric or general type.
- Remember the numeric indicator (⠼) also sets Grade 1 mode for the remainder of the symbols-sequence.

Some of the following examples of General Fractions show the choice that can be made when using the Grade 1 indicator. There is usually a preferred option. Decisions about option selection are generally associated with user and transcriber preferences, or local production formatting guidelines, and include consideration of such factors as simplicity or functionality.

Examples:

$$\frac{2/3}{6}$$

⠰⠷⠼⠃⠸⠌⠼⠉⠨⠌⠼⠋⠾

$$\frac{20+30}{40-15}$$

⠰⠷⠼⠃⠚⠐⠖⠼⠉⠚⠨⠌⠼⠙⠚⠐⠤⠼⠁⠑⠾

$$\frac{4}{8}=\frac{\square}{2}$$

⠼⠙⠌⠓ ⠐⠶ ⠰⠰⠷⠫⠼⠙⠱⠨⠌⠼⠃⠾

$$\frac{p+q}{p-q}$$

⠰⠰⠷⠏⠐⠖⠟⠨⠌⠏⠐⠤⠟⠾ preferred option

OR

⠰⠷⠏⠐⠖⠟⠨⠌⠏⠐⠤⠟⠰⠾

$$\frac{pq}{3} + \frac{p}{q}$$

⠰⠷⠏⠟⠨⠌⠼⠉⠾⠐⠖⠷⠏⠨⠌⠟⠾

$$\frac{a}{b}c$$

⠰⠰⠷⠁⠨⠌⠃⠾⠉ preferred option

OR

⠰⠷⠁⠨⠌⠃⠰⠾⠉

$$Speed = \frac{\boldsymbol{distance}}{\boldsymbol{time}}$$

The first example shown in braille below is the preferred option so as to retain more contracted words in the literary equation.

⠠⠎⠏⠑⠫ ⠐⠶ ⠰⠷⠲⠞⠨⠑⠨⠌⠐⠞⠰⠾ preferred option

OR

⠠⠎⠏⠑⠫ ⠐⠶ ⠰⠰⠷⠙⠊⠎⠞⠁⠝⠉⠑⠨⠌⠞⠊⠍⠑⠾

OR

⠰⠰⠰⠠⠎⠏⠑⠑⠙ ⠐⠶ ⠷⠙⠊⠎⠞⠁⠝⠉⠑⠨⠌⠞⠊⠍⠑⠾⠰⠄

$$\frac{x \times y}{p \times q} = \frac{xy}{pq}$$

The first option shown below is the preferred option, due to the comparison sign used in the sequences and Grade 1 indicators that are required on both sides of the equation.

⠰⠰⠰⠷⠭⠐⠦⠽⠨⠌⠏⠐⠦⠟⠾ ⠐⠶ ⠷⠭⠽⠨⠌⠏⠟⠾⠰⠄

OR

⠰⠰⠷⠭⠐⠦⠽⠨⠌⠏⠐⠦⠟⠾ ⠐⠶ ⠰⠰⠷⠭⠽⠨⠌⠏⠟⠾

OR

⠰⠷⠭⠐⠦⠽⠨⠌⠏⠐⠦⠟⠰⠾ ⠐⠶ ⠰⠷⠭⠽⠨⠌⠏⠟⠰⠾

Notes:

- It is important to remember that the opening and closing fraction indicators each have a Grade 2 (contracted) meaning. Therefore, if Grade 1 mode has not been established for some reason, then the opening and closing fraction indicators will require Grade 1 indicators.

- Remember the numeric indicator sets numeric mode as well as Grade 1 mode for the remainder of the symbols-sequence unless terminated for some reason.

Miscellaneous Symbols

Not all miscellaneous symbols are included in this training program. For a more extensive list of miscellaneous symbols and their equivalent braille representations, please refer to the *“Unified English Braille Guidelines for Technical Material”* (International Council on English Braille, 2014).

Specific use of many of the signs presented below will be outlined in further lessons.

⠩	√	open radical (root) sign, with vinculum
⠬		close radical (root) sign, (there is no print representation for this braille symbol)
⠮	∫	integral sign
⠘⠚	o	degree sign
⠶	′	foot or minute (shown as a prime sign)
⠶⠶	″	inch or second (shown as a double prime sign)
⠸⠳	|	vertical bar (absolute value)
⠼⠿	∞	infinity
⠖	!	factorial sign
⠰⠠⠡	∴	therefore sign
⠸⠪	∠	angle sign
⠼⠇	‖	parallel to
⠼⠤	⊥	perpendicular to
⠸⠿	≡	is congruent/equivalent to

⠼⠸⠸	|||	is similar to
⠨⠣	[	open square bracket
⠨⠜	]	close square bracket
⠸⠣	{	open curly bracket
⠸⠜	}	close curly bracket
⠐		dot 5 continuation indicator (positioned up close to the last character when the braille sequence is too long for the line and needs to be broken at a logical place)
⠬		visible blank space (an omission, which often occurs in fractions)
⠈⠱	/	cancelling sign
⠈⠩	✓	tick sign (not to be confused with the root sign)

Notes:

- The spacing of symbols generally follows the print.
- Some of the signs listed above have a Grade 2 (contracted) meaning. A Grade 1 indicator will therefore be necessary if the sequence is not already in Grade 1 mode.
- If the braille representation for a print sequence does not fit on the line, then a dot 5 (⠐) continuation indicator placed at a logical place immediately following the last character may be required to show the braille is continuing to the next line. Usually the preferred place to break is before a comparison sign, an operation sign or a fraction line symbol. For the purposes of this training program, take the remainder of the sequence to the margin. It is recognised there may be various local formatting guidelines.
- Remember the numeric indicator sets numeric mode as well as Grade 1 mode for the remainder of the symbols-sequence unless terminated for some reason.
- Further explanation of the use of some of these signs below will be explained in subsequent lessons.

  Examples:

  $\sqrt{a^2}$

⠰⠰⠩⠁⠔⠼⠃⠬

OR

⠰⠩⠁⠰⠔⠼⠃⠬

$\int 3^x \, dx$

⠰⠮⠼⠉⠔⠭⠙⠭

$25^\circ\mathrm{C}$

⠼⠃⠑⠘⠚⠠⠉

$10'5''$

⠼⠁⠚⠶⠼⠑⠶⠶

$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ (the dot shown as a running product)

⠼⠑⠖ ⠐⠶ ⠼⠑⠐⠲⠼⠙⠐⠲⠼⠉⠐⠲⠼⠃⠐⠲⠼⠁

$\therefore\ y = 7$

⠰⠠⠡ ⠰⠽ ⠐⠶ ⠼⠛

$\angle A + \angle B = 90^\circ$

⠸⠪⠠⠁⠐⠖⠸⠪⠠⠃ ⠐⠶ ⠼⠊⠚⠘⠚

$AB \parallel CD$

⠰⠠⠠⠁⠃ ⠼⠇ ⠰⠠⠠⠉⠙

$AB \perp CD$

⠰⠠⠠⠁⠃ ⠼⠤ ⠰⠠⠠⠉⠙

$\Delta ABC \equiv \Delta DEF$

⠰⠰⠰⠫⠼⠉⠱⠠⠠⠁⠃⠉ ⠸⠿ ⠫⠼⠉⠱⠠⠠⠙⠑⠋⠰⠠

OR

⠰⠫⠼⠉⠱⠠⠠⠁⠃⠉ ⠸⠿ ⠰⠫⠼⠉⠱⠠⠠⠙⠑⠋

$\Delta ABC\ |||\ \Delta DEF$

⠰⠰⠰⠫⠼⠉⠱⠠⠠⠁⠃⠉ ⠼⠸⠇ ⠫⠼⠉⠱⠠⠠⠙⠑⠋⠰⠄

OR

⠰⠫⠼⠉⠱⠠⠠⠁⠃⠉ ⠼⠸⠇ ⠰⠫⠼⠉⠱⠠⠠⠙⠑⠋

$$\frac{6\times3}{12}=\frac{\cancel{6}\times3}{\cancel{6}\times2}$$

Note the use of the dot 5 continuation indicator in the following example.

⠰⠯⠼⠋⠐⠦⠼⠉⠨⠌⠼⠁⠃⠾⠐

⠐⠶ ⠰⠯⠼⠋⠈⠱⠐⠦⠼⠉⠨⠌⠼⠋⠈⠱⠐⠦⠼⠃⠾

OR

Note the equals sign taken to the new line without the dot 5 continuation indicator.

⠰⠯⠼⠋⠐⠦⠼⠉⠨⠌⠼⠁⠃⠾

⠐⠶ ⠰⠯⠼⠋⠈⠱⠐⠦⠼⠉⠨⠌⠼⠋⠈⠱⠐⠦⠼⠃⠾

Yes ✓

⠠⠽⠑⠎ ⠈⠩

Superscripts and Subscripts

⠔	Level change up
⠢	Level change down
⠣	Open braille grouping sign
⠜	Closing braille grouping sign

Definition of an Item

An “item” is defined as any single symbol(s) that follows immediately after the level change indicator. It is therefore important to make clear to the reader exactly what symbol(s) must be included as a consequence of the level change indicator.

There is a defined list below of specific conditions that are considered to be the “next item”.

1. An entire number expressed in braille, i.e. the initiating numeric indicator and all succeeding symbols within the numeric mode (including any interior decimal points, commas, separator spaces, or simple numeric fraction lines).
2. An entire general fraction enclosed in general fraction indicators (⠷) and (⠾). Note that Grade 1 indicators will be required if the sequence is not already in Grade 1 mode.
3. An entire radical expression enclosed in radical indicators (such as a square root).
4. An arrow.
5. An arbitrary shape.
6. Any expression enclosed in matching pairs of round parentheses, square brackets or curly braces.

Notes:

- If none of the above conditions apply then the “item” is only the next symbol and may require braille grouping indicators, opening (⠣) and closing (⠜), to ensure the whole of the superscript or subscript has been captured. Note that Grade 1 indicators will be required for these signs if the sequence is not already in Grade 1 mode.
- Any expression when enclosed in the braille grouping indicators described above will subsequently make it clear to the reader exactly what symbols are included as part of the level change.
- A negative superscript or subscript must be enclosed in braille grouping signs because a minus sign can be an item in its own right.
- The superscript and subscript signs each have a Grade 2 (contracted) meaning so will always require Grade 1 indicators if the sequence is not already shown in Grade 1 mode.
- The numeric indicator also sets Grade 1 mode for the remainder of the symbols-sequence.

  Examples:

  2^4

  ⠼⠃⠔⠼⠙

  $x^{4.4}$

  ⠭⠰⠔⠼⠙⠲⠙

  $x^{\frac{1}{4}}$

⠭⠰⠔⠼⠁⠌⠙

$x^{\frac{a}{b}}$

⠰⠰⠭⠔⠷⠁⠨⠌⠃⠾ preferred

OR

⠭⠰⠔⠰⠷⠁⠨⠌⠃⠰⠾

$x^{(-5)}$

⠭⠰⠔⠐⠣⠐⠤⠼⠑⠐⠜

x^n

⠭⠰⠔⠝

x^{-n}

⠰⠰⠭⠔⠣⠐⠤⠝⠜

x^{-2b}

⠰⠰⠭⠔⠣⠐⠤⠼⠃⠰⠃⠜ preferred

OR

⠭⠰⠔⠰⠣⠐⠤⠼⠃⠰⠃⠜

x_{2b}

⠰⠰⠭⠢⠣⠼⠃⠰⠃⠜ preferred

OR

⠭⠰⠢⠰⠣⠼⠃⠰⠃⠜

x_n

⠭⠰⠢⠝

y_{-n}

⠰⠰⠽⠢⠣⠐⠤⠝⠜ preferred

OR

⠽⠰⠢⠰⠣⠐⠤⠝⠰⠜

$$\frac{p_1 + p_2}{p_1 - p_2}$$

⠰⠰⠷⠏⠢⠼⠁⠐⠖⠏⠢⠼⠃⠸⠌⠏⠢⠼⠁⠐⠤⠏⠢⠼⠃⠾

Simultaneous Superscripts or Subscripts

- If more than one superscript or subscript applies to the term, work from bottom to top, or left to right.

  Examples:

  $$x_1^2 = y_2^3$$

  ⠰⠰⠰⠭⠢⠼⠁⠔⠼⠃ ⠐⠶ ⠽⠢⠼⠃⠔⠼⠉⠰⠄

  $$x^2{}_k$$

  ⠰⠰⠭⠔⠼⠃⠢⠅

Left-displaced superscripts or subscripts

- When both a subscript and a superscript are shown together in print to the left of the base symbol, they are brailled by using the corresponding index expression prior to the base symbol, working from bottom to top.
- Note the Grade 1 indicator that is required in front of the subscript at the beginning of the example below but not required for the superscript sign, due to the numeric indicator setting Grade 1 mode for the remainder of that sequence.

  Example:

  $${}^{238}_{92}U$$

  ⠰⠢⠼⠊⠃⠔⠼⠃⠉⠓⠠⠥

General Note: There is often a choice of Grade 1 indicators in mathematical contexts, with use of any of the options for Grade 1 mode (symbol, word or passage) being equally correct.

Decisions about option selection are generally associated with user and transcriber preferences, or local production formatting guidelines, and include consideration for simplicity or functionality.

Therefore, for the purpose of the exercises in this training program please use the following criteria for implementing Grade 1 mode:

- Use the Grade 1 **symbol** indicator when there is only one symbol in the sequence requiring a Grade 1 indicator.
- Use the Grade 1 **word** indicator when there are two or more symbols in the sequence requiring Grade 1 mode except in a context whereby any literary elements will be affected and as a consequence will also be uncontracted.
- Use Grade 1 **passage** indicator (with Grade 1 terminator) when a comparison indicator or a space is used in the sequence and Grade 1 indicators are required on both sides of the equation. Remember, however, the impact of Grade 1 mode on any literary elements.

Exercise 1

1. $\frac{r-3}{2} \leq -6$

2. Simplify $\frac{2}{3} - \frac{x-1}{4}$

3. $\therefore x = \frac{5+\sqrt{45}}{2}$

4. $q = \sqrt[3]{x^3 + y^3 + z^3}$

5. $\frac{m^2-8}{m+1} \geq 4$

6. $\left(\frac{9}{25}\right)^{k+3} = \sqrt{\frac{3}{5}}$

7. $\frac{y}{w} > \frac{1+\sqrt{5}}{2}$

8. $\frac{42g^3h^4}{7h^2} =$

9. $S = \sqrt{\frac{3V}{h}}$

10. $A = \frac{h}{3}(d_f + 4d_m + d_l)$

Extra Exercise 1

1. $(\frac{1}{27})^{3n+1} = \frac{\sqrt{3}}{81}$

2. $A = \frac{2 - sin\theta}{2\cos\theta}$

3. $T_n = 5^n - 1$

4. $x = t\sqrt{49 - t^2}$

5. $\log\left(\frac{x^2}{y}\right) = \log 2$

6. $(x - h)^2 = \pm 4a(y - k)$

7. Let $r = \sqrt{\frac{4p}{3}}$

8. $\frac{2b}{5} - \frac{1}{2} \geq 6$

9. $\frac{q-2}{3} < 2 + \frac{3q}{4}$

10. $(x + \frac{1}{x})^2$

Lesson 2 - Functions

A technical definition of a function is: a **relation** from a set of inputs to a set of possible outputs where each input is related to exactly one output.

Follow the print for the spelling and capitalisation of function names.

Where letters before or after the function name are written in italics to indicate they are variables, the italics should be omitted in braille.

Example:

cos*C*

⠉⠕⠎⠠⠉

Trigonometric functions

Common trigonometric functions are Sine, Cosine, Tangent, Secant, Cosecant and Cotangent. These functions are usually abbreviated in print as sin, cos, tan, sec, cosec and cot.

- Sine (Sin), Cosine, Tangent and Cotangent may be contracted unless already in Grade 1 mode.

  Example:

  Cosine C

  ⠠⠉⠕⠎⠔⠑ ⠰⠠⠉

- Where a function name is **preceded** or **followed** by a number, then the number should be written unspaced from the function name.
- Remember the numeric indicator also sets Grade 1 mode for the next symbols-sequence.

  Examples:

  5tan30°

  ⠼⠑⠞⠁⠝⠼⠉⠚⠘⠚

  Cos15

  ⠠⠉⠕⠎⠼⠁⠑

  2sin8*x*

  ⠼⠃⠎⠊⠝⠼⠓⠭

- Insert a space if a function name is **preceded** directly by a lower or upper-case Latin letter, with no intervening braille indicators or brackets.

  Examples:

  *x*cos 30

  ⠰⠭ ⠉⠕⠎⠼⠉⠚

  Xcos 30

  ⠰⠠⠭ ⠉⠕⠎⠼⠉⠚

- Insert a space if a function name is **followed** directly by a lower-case letter with no intervening braille indicators or brackets.

  Example:

  log *x*

  ⠇⠕⠛ ⠰⠭

- The space is not needed if the function name is already separated by a bracket or by a braille indicator, such as a capitalisation indicator, a Greek letter indicator or an opening general fraction indicator.
- Remember the opening and closing general fraction indicators have a Grade 2 (contracted) meaning so unless you are already in Grade 1 mode, a Grade 1 symbol indicator will be required before the opening and closing fraction indicator.
- We will investigate Greek letters more thoroughly in Lesson 5 of this Training Manual.

  Examples:

  log $(x+y)$

  ⠇⠕⠛⠐⠣⠭⠐⠖⠽⠐⠜

  $\cos B$

  ⠉⠕⠎⠠⠃

  $\mathrm{Tan}\theta$

  ⠞⠁⠝⠨⠹

  tan $\frac{\boldsymbol{\theta}}{\mathbf{2}}$

  ⠞⠁⠝⠰⠷⠨⠹⠨⠌⠼⠃⠾

Inverse trigonometric functions

The examples of the inverse functions below have been shown in Grade 1 word mode, as a Grade 1 indicator is required in front of the superscript sign and the opening braille grouping indicator showing the whole of the “next item”.

The inverse functions may be written as:

⠰⠰⠎⠊⠝⠔⠣⠐⠤⠼⠁⠜	$\sin^{-1}$
⠰⠰⠉⠕⠎⠔⠣⠐⠤⠼⠁⠜	$\cos^{-1}$
⠰⠰⠞⠁⠝⠔⠣⠐⠤⠼⠁⠜	$\tan^{-1}$
⠰⠰⠎⠑⠉⠔⠣⠐⠤⠼⠁⠜	$\sec^{-1}$
⠰⠰⠉⠕⠎⠑⠉⠔⠣⠐⠤⠼⠁⠜	cosec^{-1}
⠰⠰⠉⠕⠞⠔⠣⠐⠤⠼⠁⠜	$\cot^{-1}$

Logarithmic functions

- The logarithmic function is usually written as log or Log and may be followed by a subscript indicating the base.

  Example:

  $$log_a x = \frac{log_b x}{log_b a}$$

  ⠰⠰⠰⠇⠕⠛⠢⠁⠭ ⠐⠶ ⠷⠇⠕⠛⠢⠃⠭⠨⠌⠇⠕⠛⠢⠃⠁⠾⠰⠄

- A logarithm to base “e” is called a natural log and is often abbreviated to ln.

  Example:

  $\ln e = 15$

  ⠇⠝ ⠰⠑ ⠐⠶ ⠼⠁⠑

The Limit functions

Limit, lim, lm, lt are all used to indicate limit, sometimes with capitals, sometimes without.

Modifiers when shown directly above and directly below the character will be addressed in Lesson 3 and Lesson 4 of this Training Manual.

Example:

Lim f(x) = 1

⠠⠇⠊⠍ ⠋⠐⠣⠭⠐⠜ ⠐⠶ ⠼⠁

Remember: As referred to in the General Note in Lesson 1, there is often a choice of Grade 1 indicators in mathematical contexts, with any of the options of Grade 1 mode (symbol, word or passage) being equally correct.

Decisions about option selection are generally associated with user and transcriber preferences, or local production formatting guidelines, and include consideration for simplicity or functionality.

Therefore, for the purpose of the exercises in this training program, please use the following criteria for implementing Grade 1 mode:

- Use the Grade 1 **symbol** indicator when there is only one symbol in the sequence requiring a Grade 1 indicator.
- Use the Grade 1 **word** indicator when there are two or more symbols in the sequence requiring Grade 1 mode except in a context whereby any literary elements will be affected and as a consequence will also be uncontracted.
- Use Grade 1 **passage** indicator (with Grade 1 terminator) when a comparison indicator or a space is used in the sequence and Grade 1 indicators are required on both sides of the equation. Remember however, the impact of Grade 1 mode on any literary elements.

Hint:

- The numeric indicator also sets Grade 1 mode for the remainder of the symbols-sequence.

Exercise 2

1. $\text{Log}\frac{y}{4}$

2. $-\frac{1}{4}\sec^4 x$

3. $\text{Log}_2 \frac{\sqrt{2}}{4}$

4. $\theta = \cos^{-1}(\frac{5.8}{7.3})$

5. $\ln 9.8 + \log_{10} 17$

6. $\log_a(\frac{x}{y})$

7. $\theta = n\pi + \tan^{-1} a$

8. $\log_a x = \frac{\log_b x}{\log_b a}$

9. $\cot A = \frac{\cos A}{\sin A}$

10. $\sec A = \frac{1}{\cos A}$

Review Exercise 2

1. $a^{3y-5} = \frac{1}{a^2}$

2. $\ln(4x + 5)$

3. $y - y_1 = m(x - x_1)$

4. $\sin^{-1} \frac{x}{a} + C$

5. $\log_a x^n = n \log_a x$

6. $K = n\sqrt{\frac{a}{p}}$

7. $\log_a \sqrt{xy}$

8. $f(x) = 3x^4 + 4x^3 - 12x^2$

9. $\log\sqrt{x^2 - 4x + 4} - \log(x - 2)$

10. $\frac{-5 \pm \sqrt{21}}{2}$

Lesson 3 - Modifiers, Bars and Dots

Modifiers

Modifiers directly above or below

If an item is written in print directly above or directly below a term, rather than to the right or left, use the directly above indicator or directly below indicator instead of the superscript or subscript. Further explanation of a Modifier directly above or below is explained in Lesson 4.

As stated below in this lesson, special signs are provided for bars and a dot directly above and directly below.

⠨⠔	Expression directly above
⠨⠢	Expression directly below

Definition of an Item

An “item” is defined as any single symbol(s) that follows immediately after the level change indicator. It is therefore important to make clear to the braille reader exactly what symbol(s) must be included as a consequence of the level change indicator.

Below is a defined list of specific conditions that are considered to be the “next item”:

- An entire number expressed in braille, i.e., the initiating numeric indicator and all succeeding symbols within the numeric mode (including any interior decimal points, commas, separator spaces, or simple numeric fraction lines).
- An entire general fraction enclosed in general fraction indicators (⠷) and (⠾). Note that Grade 1 indicators will be required if the sequence is not already in Grade 1 mode.
- An entire radical expression enclosed in radical indicators (such as a square root).
- An arrow.
- An arbitrary shape.
- Any expression enclosed in matching pairs of round parentheses, square brackets, or curly braces.

Notes:

- If none of the above conditions apply, then the “item” is the only the next symbol and may require braille grouping indicators, opening (⠣) and closing (⠜), to ensure the whole of the superscript or subscript has been captured. Note that

Grade 1 indicators will be required for these signs if the sequence is not already in Grade 1 mode.

- Any expression when enclosed in the braille grouping indicators (described below) will subsequently make it clear to the reader exactly what symbols are included as part of the level change.
- The superscript (⠔) and subscript (⠢) signs each have a Grade 2 (contracted) meaning so will always require Grade 1 indicators if the sequence is not already shown in Grade 1 mode.
- A negative superscript or subscript must be enclosed in braille grouping signs because a minus sign can be an item in its own right.
- The numeric indicator also sets Grade 1 mode for the remainder of the symbols-sequence.

Braille Grouping Symbols

Refer to the definition of an item above to determine if the braille grouping symbols will be required.

⠣	Opening braille grouping sign
⠜	Closing braille grouping sign

Bars

Common modifiers such as bars are treated separately from something that is written directly above or directly below a term.

⠱	bar over previous item
⠠⠱	bar under previous item

Note:

- The bar directly over a previous item (⠱) and the bar directly under a previous item (⠠⠱) have a Grade 2 (contracted) meaning, so unless you are already in Grade 1 mode, a Grade 1 symbol indicator will be required before the bar over and bar under indicators.

Examples:

$\overline{x}$ bar over

⠭⠰⠱

$\bar{x} = \frac{10+11+12}{3}$ bar over

⠰⠰⠰⠭⠱ ⠐⠶ ⠷⠼⠁⠚⠐⠖⠼⠁⠁⠐⠖⠼⠁⠃⠨⠌⠼⠉⠾⠰⠄

OR

⠭⠰⠱ ⠐⠶ ⠰⠷⠼⠁⠚⠐⠖⠼⠁⠁⠐⠖⠼⠁⠃⠨⠌⠼⠉⠾

$\underline{x + y}$ bar under

⠰⠰⠣⠭⠐⠖⠽⠜⠠⠱ preferred option as there is more than one Grade 1 indicator required in the sequence

OR

⠰⠣⠭⠐⠖⠽⠰⠜⠰⠠⠱

Two indicator bars applied to the same item

If two indicators apply to the same item, then braille grouping symbols must be used to show which applies first.

Examples:

$x^{\bar{y}}$

⠰⠰⠭⠔⠣⠽⠱⠜

$\overline{x^{y}}$

⠰⠰⠣⠭⠔⠽⠜⠱

Dots

Common modifiers such as a single dot are treated separately from something that is written directly above or directly below a term.

⠘⠲ dot over a previous item: the recurring decimal and single derivative

Recurring decimal:

⠘⠲ dot over a previous item: the recurring decimal

The recurring decimal (shown below with the dot over the digit/s recurring) will require braille grouping signs opening (⠣) and closing (⠜) otherwise the dot would refer to the entire number.

Examples:

$5.\dot{5}$

⠼⠑⠲⠣⠼⠑⠜⠘⠲

$0.57\dot{1}2\dot{3}$

⠼⠚⠲⠑⠛⠣⠼⠁⠜⠘⠲⠼⠃⠣⠼⠉⠜⠘⠲

OR

because the digits 123 in the above example are being implicitly grouped in print as recurring, this can also be brailled as:

$0.57\dot{1}2\dot{3}$

⠼⠚⠲⠑⠛⠣⠼⠁⠃⠉⠜⠘⠲

Derivatives of $\dot{x}$

The derivative of $\dot{x}$ has its own sign as indicated below.

⠘⠲ dot over a previous item: the single derivative

Example:

$\dot{x}$

⠭⠘⠲

Derivatives of $\ddot{x}$

The x double dot ($\ddot{x}$) sign is treated differently to the single derivative of $\dot{x}$ which has its own specific braille sign as explained above.

⠲⠲ dots over a previous item: the double derivative

The x double dot $(\ddot{x})$ will require the signs for directly above ⠨⠔ or directly below ⠨⠢ a term as stated at the beginning of this lesson.

Derivatives of x double dot $(\ddot{x})$, will also require braille grouping signs, opening (⠣) and closing (⠜), to ensure the effect of the whole of the double dot is captured.

Example:

$\ddot{x}$

⠰⠰⠭⠨⠔⠣⠲⠲⠜

Notes:

- The braille grouping signs have a Grade 2 (contracted) meaning. Therefore, unless you are already in Grade 1 mode, Grade 1 symbol indicators will be required before the opening and closing braille grouping signs.
- The numeric indicator also sets Grade 1 mode for the remainder of the next symbols-sequence.
- If the braille representation for a print sequence does not fit on the line, then a dot 5 (⠐) continuation indicator placed at a logical place immediately following the last character may be required to show the braille is continuing to the next line. Usually the preferred place to break is before a comparison sign, an operation sign, or a fraction line symbol. For the purposes of this training program, take the remainder of the sequence to the margin as it is recognised there may be various local formatting guidelines.

Remember:

As referred to in the General Note in Lesson 1, there is often a choice of Grade 1 indicators in mathematical contexts, with any of the options of Grade 1 mode (symbol, word, or passage) being equally correct.

Decisions about option selection are generally associated with user and transcriber preferences, or local production formatting guidelines, and include consideration for simplicity or functionality.

Therefore, for the purpose of the exercises in this training program, please use the following criteria for implementing Grade 1 mode:

- Use the Grade 1 **symbol** indicator when there is only one symbol in the sequence requiring a Grade 1 indicator.
- Use the Grade 1 **word** indicator when there are two or more symbols in the sequence requiring Grade 1 mode except in a context whereby any literary elements will be affected and as a consequence will also be uncontracted.

- Use Grade 1 **passage** indicator (with Grade 1 terminator) when a comparison indicator or a space is used in the sequence and Grade 1 indicators are required on both sides of the equation. Remember however, the impact of Grade 1 mode on any literary elements.

Hint:

- The numeric indicator also sets Grade 1 mode for the remainder of the symbols-sequence.

Exercise 3

1. $z = \frac{x-\bar{x}}{s}$

2. $\bar{x} = \frac{\text{sum of scores}}{\text{number of scores}}$

3. $y^2 = -i\bar{y}$

4. $\bar{x} = \frac{25 + 35 + 45}{5}$

5. $\underline{PQ}$

6. $\underline{m + n}$

7. $4.1\dot{5}2\dot{5}$

8. $\dot{x} = \frac{t}{t^2 + 3} ms^{-1}$

9. $\ddot{x} = g - kv$

10. $\ddot{x} = 25e^{5t}\, ms^{-2}$

Review Exercise 3

1. $\log_x(\frac{1}{3}) = -1$

2. $\therefore \theta = 37^\circ 23'$

3. $\frac{n^2 - n - 6}{n^2 + 5n + 6}$

4. $\sqrt{\frac{-3c}{b - a}}$

5. $\log_a x = \frac{\log_b x}{\log_b a}$

6. $\log_7 \sqrt[4]{7}$

7. $\therefore\ \Delta\mathrm{RST} \equiv \Delta\mathrm{XYZ}$

8. $y = 2\sqrt{x} = 2x^{\frac{1}{2}}$

9. $d = \sqrt{(x_2 - x_1)^2}$

10. $\frac{\log_e x}{e^{2x} + x}$

Lesson 4 - Modifiers, Arrows and Limits

Modifiers

Modifiers directly above or below

If something is written directly above or directly below a term rather than to the right or left, use the directly above indicator or directly below indicator instead of the superscript or subscript.

⠨⠔	Expression directly above
⠨⠢	Expression directly below

Comparison signs are usually unspaced when they appear in an expression placed directly above or directly below the character.

However, a common modifier such as an arrow is treated similar to bars and a dot (described in Lesson 3) and has its **own** sign instead of using the indicators for “directly above” or “directly below”. This is introduced below in this lesson.

Definition of an Item

An “item” is defined as any single symbol(s) that follows immediately after the level change indicator. It is therefore important to make clear to the braille reader exactly what symbol(s) must be included as a consequence of the level change indicator.

Below is a defined list of specific conditions that are considered to be the “next item”:

- An entire number expressed in braille, i.e. the initiating numeric indicator and all succeeding symbols within the numeric mode (including any interior decimal points, commas, separator spaces, or simple numeric fraction lines).
- An entire general fraction enclosed in general fraction indicators (⠷) and (⠾). Note that Grade 1 indicators will be required if the sequence is not already in Grade 1 mode.
- An entire radical expression enclosed in radical indicators (such as a square root).
- An arrow.
- An arbitrary shape.
- Any expression enclosed in matching pairs of round parentheses, square brackets, or curly braces.

Notes:

- If none of the above conditions apply then the "item" is the only the next symbol and may require braille grouping indicators opening (⠣) and closing (⠜) to ensure the whole of the superscript or subscript has been captured. Note that Grade 1 indicators will be required for these signs if the sequence is not already in Grade 1 mode.
- Any expression when enclosed in the braille grouping indicators described below will subsequently make it clear to the reader exactly what symbols are included as part of the level change.

Braille Grouping Symbols

Refer to the Definition of an Item above to determine if the braille grouping symbols shown below will be required.

⠣	Opening braille grouping sign
⠜	Closing braille grouping sign

Example:

The example below using capital Greek sigma (Σ) shows an expression directly above and directly below a term with the use of a comparison sign, which is unspaced when placed directly above or below a term.

$$\sum_{n=2}^{5} n^2$$

⠰⠰⠠⠨⠎⠨⠢⠣⠝⠐⠶⠼⠃⠜⠨⠔⠼⠑⠝⠔⠼⠃

OR

⠠⠨⠎⠨⠢⠰⠣⠝⠐⠶⠼⠃⠜⠨⠔⠼⠑⠝⠔⠼⠃

Arrows

For a complete list of Arrows, refer to the *Unified English Braille Guidelines for Technical Material* (International Council on English Braille, 2014).

The arrow indicator has a Grade 2 (contracted) meaning so unless you are already in Grade 1 mode, a Grade 1 symbol indicator will be required before the arrow indicator.

⠳		arrow indicator
⠳⠕	→	simple right pointing arrow (east)
⠳⠪	←	simple left pointing arrow (west)

Simple Arrows

- A simple arrow has a standard barbed tip at one end (like a print letter “v” on its side) pointing away from the shaft.
- The shaft is straight, and its length and thickness are not significant in braille.
- Simple arrows are represented by an arrow indicator as shown in the list above.
- Arrows are considered signs of comparison and should usually be spaced, with the exception of when they are written immediately below an item, as in Limit Functions described later in this lesson.

  Examples:

  $\mathrm{n} \to 0$ (n tends to zero shown using Grade 1 symbol mode)

  ⠰⠝ ⠰⠳⠕ ⠼⠚

  OR

  $\mathrm{n} \to 0$ (n tends to zero shown using Grade 1 passage mode)

  ⠰⠰⠰⠝ ⠳⠕ ⠼⠚⠰⠄

  $x \to \infty$ (*x* tends to infinity shown using Grade 1 symbol mode)

  ⠰⠭ ⠰⠳⠕ ⠼⠿

  OR

  $x \to \infty$ (*x* tends to infinity shown using Grade 1 passage mode)

⠰⠰⠰⠭ ⠳⠕ ⠼⠿⠰⠄

Arrow symbol directly above and below

Common modifiers such as arrows are treated similar to bars and a single dot as described in Lesson 3 and have their **own** sign instead of using the indicators for “directly above” or “directly below” as shown above in this lesson.

Only use these arrow indicators when a simple right pointing arrow is the only modifier directly above or directly below an item.

⠘⠱ Arrow directly above

⠠⠘⠱ Arrow directly below

The symbol shown above for a simple right-pointing arrow directly above or directly below an item has a Grade 2 (contracted) meaning, so unless you are already in Grade 1 mode, a Grade 1 symbol indicator will be required before the arrow indicator.

Examples:

⠭⠰⠘⠱ $\vec{x}$ Simple right-pointing arrow over previous item (x)

⠭⠰⠠⠘⠱ $\underset{\rightarrow}{x}$ Simple right-pointing arrow under previous item (x)

Notes:

- The use of braille grouping indicators in the example below make it clear to the reader exactly what symbols are affected by the arrow under the previous item (an arrow under the H_2).
- The example below has been placed in Grade 1 word mode due to more than one Grade 1 symbol indicator being required in the sequence.

  Example:

  ⠰⠰⠣⠠⠓⠢⠼⠃⠜⠠⠘⠱ $\underset{\rightarrow}{H_2}$

The Limit Function

Limit, lim, lm, lt are all used to indicate limit, sometimes with capitals, sometimes without.

- Signs of operation and comparison are unspaced when they appear in an expression which is not on the base line, such as directly above or directly below the character.

  Example:

  $$\underset{x\to a}{\mathrm{Lim}}\, f(x)$$

  ⠰⠰⠠⠇⠊⠍⠨⠢⠣⠭⠳⠕⠁⠜⠋⠐⠣⠭⠐⠜

Notes:

- The example below contains a General Fraction.
- The example below has been placed in Grade 1 word mode due to more than one Grade 1 symbol indicator being required in the sequence.

  Example:

  $$\lim_{x\to 0}\frac{x^2\ +3x}{x}$$

  ⠰⠰⠇⠊⠍⠨⠢⠣⠭⠳⠕⠼⠚⠜⠷⠭⠔⠼⠃⠐⠖⠼⠉⠭⠨⠌⠭⠾

Remember:

As referred to in the General Note in Lesson 1, there is often a choice of Grade 1 indicator, with any of the options of Grade 1 mode (symbol, word, or passage) being equally correct.

Decisions about option selection are generally associated with user and transcriber preferences, or local production formatting guidelines, and include consideration for simplicity or functionality.

Therefore, for the purpose of the exercises in UEB Mathematics Online, use the following criteria for implementing Grade 1 mode:

- Use the Grade 1 **symbol** indicator when there is only one symbol in the sequence requiring a Grade 1 indicator.
- Use the Grade 1 **word** indicator when there are two or more symbols in the sequence requiring Grade 1 mode except in a context whereby any literary elements will be affected and as a consequence will also be uncontracted.
- Use Grade 1 **passage** indicator (with Grade 1 terminator) when a comparison indicator or a space is used in the sequence and Grade 1 indicators are required on both sides of the equation. Remember however, the impact of Grade 1 mode on any literary elements.

Hints:

- The numeric indicator also sets Grade 1 mode for the remainder of the symbols-sequence.
- Insert a space if a function name is **preceded** directly by a lower or upper-case Latin letter, with no intervening braille indicators or brackets.

Exercise 4

1. Input → process → output

2. $x \rightarrow y$

3. $\overrightarrow{AB}$

4. $\overrightarrow{AB} + \overrightarrow{CD}$

5. $\lim\limits_{x \rightarrow 4} x^2 + 5$

6. $\lim\limits_{x \rightarrow \infty} \frac{x}{x^2}$

7. $\lim\limits_{g \rightarrow 1} \frac{g^2 - 1}{g - 1}$

8. $\lim\limits_{t \rightarrow 4} \frac{t^2 - 16}{t - 4}$

9. $\lim\limits_{x \rightarrow \infty} \frac{3x^2}{x^2 - 2x}$

10. $\lim\limits_{h \rightarrow 0} \frac{2h^2x + hx^2}{h}$

Review Exercise 4

1. $a \rightarrow y = \ln(x+1)$

2. $\frac{\sqrt{2x+1}}{(x-3)^3}$

3. $\frac{2x}{(x+5)^{\frac{1}{2}}}$

4. $OC = \sqrt{r^2 - \frac{x}{2}}$

5. $c^2 = a^2 + b^2 - 2ab\cos C$

6. $\frac{48p^2q^4}{6pq^3}$

7. $A = \pi(R^2 - r^2)$

8. $n = \frac{42 \sin 117^\circ}{\sin 35^\circ}$

9. $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

10. $\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$

Lesson 5 - Greek Letters (continued) and Probability

Greek Letters

Greek letters are used extensively in Mathematics. While only a small number have been used in this Training Manual, the principles for use remain the same for all. Refer to the *Unified English Braille Guidelines for Technical Material* (International Council on English Braille, 2014) for a complete list.

⠨⠁	α	alpha (lower case)	⠠⠨⠁	Α	Capital
⠨⠃	β	beta (lower case)	⠠⠨⠃	Β	Capital
⠨⠙	δ	delta (lower case)	⠠⠨⠙	Δ	Capital
⠨⠑	ε	epsilon (lower case)	⠠⠨⠑	Ε	Capital
⠨⠛	γ	gamma (lower case)	⠠⠨⠛	Γ	Capital
⠨⠹	θ	theta (lower case)	⠠⠨⠹	Θ	Capital
⠨⠇	λ	lambda (lower case)	⠠⠨⠇	Λ	Capital
⠨⠍	μ	mu (lower case)	⠠⠨⠍	Μ	Capital
⠨⠏	π	pi (lower case)	⠠⠨⠏	Π	Capital
⠨⠎	σ	sigma (lower case)	⠠⠨⠎	Σ	Capital
⠨⠋	ϕ	phi (lower case)	⠠⠨⠋	Φ	Capital

Examples:

$$\tan\theta = \frac{\sqrt{5}}{2}$$

⠞⠁⠝⠨⠹ ⠐⠶ ⠰⠰⠷⠩⠼⠑⠬⠨⠌⠼⠃⠾ preferred

OR

⠞⠁⠝⠨⠹ ⠐⠶ ⠰⠷⠰⠩⠼⠑⠬⠨⠌⠼⠃⠾

$$\alpha + \ \beta \ + \gamma = -\frac{b}{a}$$

⠨⠁⠐⠖⠨⠃⠐⠖⠨⠛ ⠐⠶ ⠐⠤⠰⠰⠷⠃⠨⠌⠁⠾ preferred

OR

⠨⠁⠐⠖⠨⠃⠐⠖⠨⠛ ⠐⠶ ⠐⠤⠰⠷⠃⠨⠌⠁⠰⠾

$$\mu(n) = 1$$

⠨⠍⠐⠣⠝⠐⠜ ⠐⠶ ⠼⠁

$$V = \ \frac{4}{3}\pi r^3$$

⠰⠠⠧ ⠐⠶ ⠼⠙⠌⠉⠨⠏⠗⠔⠼⠉

$$\Delta = b^2 \ - 4ac$$

⠠⠨⠙ ⠐⠶ ⠃⠰⠔⠼⠃⠐⠤⠼⠙⠰⠁⠉

Notes:

- Remember if something is written directly above or directly below a term rather than to the right or left, use the directly above indicator (⠨⠔) or directly below indicator (⠨⠢) instead of the superscript or subscript indicator.
- Care must be taken to ensure that the sign for capital delta (Δ) is not confused with a triangle sign.
- Any expression when enclosed in the braille grouping indicators, opening (⠣) and closing (⠜), will subsequently make it clear to the reader exactly what symbols are included as part of the level change.
- Remember that Grade 1 indicators will be required for the braille grouping symbols if the sequence is not already in Grade 1 mode.
- It is also preferable to unspace a comparison sign when the comparison sign is not on the base line such as in sigma notation.

  Example:

  $$\sum_{x\ =1}^{n} x_{i^2}$$

⠠⠨⠎⠨⠢⠰⠣⠭⠐⠶⠼⠁⠜⠨⠔⠝⠭⠢⠊⠔⠼⠃

Probability

Refer to the "*Unified English Braille Guidelines for Technical Material*" (International Council on English Braille, 2014) for further representations of Set Theory, Group Theory and Logic.

The following signs may be used in the topic of Probability at the level of Mathematics being presented in this training program.

⠨⠖	∪	union (upright ∪ shape): unspaced
⠨⠦	∩	intersection (inverted ∩ shape): unspaced
⠘⠑	∈	is an element of: spaced
⠘⠣	⊂	is a subset of: spaced

Examples:

$A \cap B \ \subset \ A \cup B$ (A intersection B is a subset of A union B)

⠠⠁⠨⠦⠠⠃ ⠘⠣ ⠠⠁⠨⠖⠠⠃

$3 \ \in \ A \cap B$ (3 is an element of A intersection B)

⠼⠉ ⠘⠑ ⠠⠁⠨⠦⠠⠃

Probability is the study of how likely it is that something will happen.

Probability is when there is some doubt about the outcome, but the degree of doubt is different. Probability is about the doubt: it is the study of events that may or may not happen, rather than of events that will happen or that have already happened.

The probability of an outcome is defined as the ratio of the number of favourable outcomes to the number of possible outcomes, assuming that the outcomes are equally likely. Thus, the probability of P(A) of a particular result A is:

$$P(A) = \frac{number\ of\ favourable\ outcomes}{number\ of\ possible\ outcomes}$$

⠠⠏⠐⠣⠠⠁⠐⠜ ⠐⠶ ⠰⠷⠝⠥⠍⠃⠻ ⠷ ⠋⠁⠧⠳⠗⠁⠃⠇⠑

⠳⠞⠉⠕⠍⠑⠎⠨⠌⠝⠥⠍⠃⠻ ⠷ ⠏⠕⠎⠎⠊⠃⠇⠑

⠳⠞⠉⠕⠍⠑⠎⠰⠾

OR

$$P(\text{Event}) = \frac{number\ of\ ways\ Event\ can\ occur}{number\ of\ possible\ outcomes}$$

⠠⠏⠐⠣⠠⠑⠧⠢⠞⠐⠜ ⠐⠶ ⠰⠷⠝⠥⠍⠃⠻ ⠷ ⠺⠁⠽⠎

⠠⠑⠧⠢⠞ ⠉ ⠕⠒⠥⠗⠨⠌⠝⠥⠍⠃⠻ ⠷ ⠏⠕⠎⠎⠊⠃⠇⠑

⠳⠞⠉⠕⠍⠑⠎⠰⠾

Note:

The example shown above has been brailled using Grade 1 symbol indicators where required rather than using Grade 1 word or Grade 1 passage mode due the number of literary elements in the sequence.

Venn Diagrams

Venn diagrams are a visual (or graphical) way to represent sets of data. You can name the sets with letters and talk about various properties of the sets.

Venn diagrams are an efficient tool to be used when analysing data to find the probability of events.

While it is not critical to understand the mathematics of Probability and Venn diagrams when transcribing braille, it may be helpful for the brief explanation below that was extracted from a student's textbook:

Example:

A description of a Venn diagram is as follows:

There is an outer group in a rectangle containing all the members of a set S, an inner group positioned in a circle on the left within the rectangle containing all the members of set D, an inner group positioned in a circle on the right within the rectangle containing all the members of set F.

The area where the inner groups (circles) overlap contains the members that are in both sets D and F. The region outside the groups of D and F contain all the members of set S that are in neither set D nor set F.

The set made by combining sets D and F is called **union** of the two sets, written as D ∪ F or ⠠⠙⠨⠖⠠⠋ in braille. It consists of all the elements that are in D or in F or in both D and F.

The set made up of the elements common to sets D and F is called the **intersection** of the two sets, written as D ∩ F or ⠠⠙⠨⠦⠠⠋ in braille. The intersection contains numbers that belong to both sets.

Complementary Events

The **complement of event** A is the event "A **does not occur"** and can be denoted by $\bar{A}$.

If an experiment has n possible outcomes, m of which are associated with event A, then $(n - m)$ outcomes are associated with event $\bar{A}$.

Examples:

$$P(A) = \frac{m}{n}$$

⠠⠏⠐⠣⠠⠁⠐⠜ ⠐⠶ ⠰⠰⠷⠍⠨⠌⠝⠾

OR

⠠⠏⠐⠣⠠⠁⠐⠜ ⠐⠶ ⠰⠷⠍⠨⠌⠝⠰⠾

AND

$$P(\bar{A}) = \frac{n-m}{n}$$

⠰⠰⠰⠠⠏⠐⠣⠠⠁⠱⠐⠜ ⠐⠶ ⠷⠝⠐⠤⠍⠨⠌⠝⠾⠰⠄

⠠⠏⠐⠣⠠⠁⠰⠱⠐⠜ ⠐⠶ ⠰⠰⠷⠝⠐⠤⠍⠨⠌⠝⠾

⠠⠏⠐⠣⠠⠁⠰⠱⠐⠜ ⠐⠶ ⠰⠷⠝⠐⠤⠍⠨⠌⠝⠰⠾

Independent Events

Two events are **independent** if the outcome of each event is not affected by the outcome of the other event. This means that the probability of both events together is the same as the probability of both events separately, and so their probabilities can be multiplied together as if they are separate successful outcomes. In other words:

Example:

If events A and B are independent, then:

$$\mathrm{P}(\mathrm{A\ and\ B}) = \mathrm{P}(\mathrm{A}) \times \mathrm{P}(\mathrm{B})$$

⠠⠏⠐⠣⠠⠁ ⠯ ⠠⠃⠐⠜ ⠐⠶

⠠⠏⠐⠣⠠⠁⠐⠜⠐⠦⠠⠏⠐⠣⠠⠃⠐⠜

Dependent Events

Two events are **dependent** if the outcome of one event is affected by the outcome of the other event. This means that the probability of both events together is ***not*** the same as the probability of both events separately. Their probabilities cannot be simply multiplied together as if they are separate successive outcomes (as for independent events). In other words:

Example:

If events A and B are dependent, then:

$$\mathrm{P}(\mathrm{A}\ \mathrm{and}\ \mathrm{B}) \neq \mathrm{P}(\mathrm{A}) \times \ \mathrm{P}(\mathrm{B})$$

⠠⠏⠐⠣⠠⠁ ⠯ ⠠⠃⠐⠜ ⠐⠶⠈⠱

⠠⠏⠐⠣⠠⠁⠐⠜⠐⠦⠠⠏⠐⠣⠠⠃⠐⠜

Remember:

As referred to in the General Note in Lesson 1, there is often a choice of Grade 1 indicator, with any of the options of Grade 1 mode (symbol, word, or passage) being equally correct.

Decisions about option selection are generally associated with user and transcriber preferences, or local production formatting guidelines, and include consideration for simplicity or functionality.

Therefore, for the purpose of the exercises in UEB Mathematics Online, use the following criteria for implementing Grade 1 mode:

- Use the Grade 1 **symbol** indicator when there is only one symbol in the sequence requiring a Grade 1 indicator.
- Use the Grade 1 **word** indicator when there are two or more symbols in the sequence requiring Grade 1 mode except in a context whereby any literary elements will be affected and as a consequence will also be uncontracted.
- Use Grade 1 **passage** indicator (with Grade 1 terminator) when a comparison indicator or a space is used in the sequence and Grade 1 indicators are required on both sides of the equation. Remember however, the impact of Grade 1 mode on any literary elements.

Hints:

- The numeric indicator also sets Grade 1 mode for the remainder of the symbols-sequence.
- If the braille representation for a print sequence does not fit on the line, then a dot 5 (⠐) continuation indicator placed at a logical place immediately following the last character may be required to show the braille is continuing to the next line. Usually the preferred place to break is before a comparison sign, an operation sign, or a fraction line indicator. For the purposes of this training program take the remainder of the sequence to the margin as it is recognised there may be various local formatting guidelines.

- Signs of operation and comparison are unspaced when they appear in an expression which is not on the base line such as directly above or directly below the character.

Exercise 5

1. $A = \frac{\theta}{360} \times \pi r^2$

2. $z = \frac{x-\mu}{\sigma}$

3. $(\cos\frac{\pi}{4} - \tan\frac{\pi}{4})^2$

4. $\theta = 2n\pi \pm cos^{-1} a$

5. $\tan(\theta + \phi) = \frac{\tan\theta + \phi}{1 - \tan\theta\tan\phi}$

6. $n(V \cup C) = n(V) + n(C)$

7. $X \cap Y = \{1, 11, 12\}$

8. $n(D \cup F) = n(C) + n(F) - n(D \cap F)$

9. $P(\bar{A} \text{ or } B) = 1$

10. $n(E \cap F) = 0$

Review Exercise 5

1. $\frac{pq}{5} \times \frac{p}{q}$

2. Roots of $P(z)$ are $\alpha, \overline{\alpha}, \beta, \bar{\beta}$

3. $\sin\theta = \frac{o}{h}$

4. $\overline{x} = \frac{\Sigma f x}{\Sigma f}$

5. $\lim_{x \to \infty} \frac{1}{x} = 0$

6. $\lim_{\delta \to 0} \frac{\delta y}{\delta x}$

7. $\frac{y+1}{y-1} > 2$

8. $\sum_{x=1}^{n}(2^r x)$

9. $\sin(\pi - \frac{\pi}{6})$

10. $\frac{1}{2}[cos\ (A) + \cos\ (B)]$

Lesson 6 - Calculus: Differentiation

As Calculus is a defined topic in senior secondary level mathematics it has been introduced specifically into the lessons for this training program. The same rules and principles of braille transcription should be observed that have been introduced previously.

There are two main branches of Calculus: Differentiation and Integration. The topic of Integration will be introduced in Lesson 7.

Differentiation is used to calculate the rate at which two variables change in relation to one another.

The process of finding the gradient of a tangent is called **differentiation.** The resulting function is called the **derivative.**

Examples:

If *y* = *f(x)* then the derivative is $\frac{dy}{dx}$ or $f'(x)$ and the partial derivative is $\frac{\delta y}{\delta x}$.

⠠⠊⠋ ⠰⠽ ⠐⠶ ⠋⠐⠣⠭⠐⠜ ⠮⠝ ⠮ ⠙⠻⠊⠧⠁⠞⠊⠧⠑
⠊⠎ ⠰⠷⠙⠽⠨⠌⠙⠭⠰⠾ ⠕⠗ ⠋⠶⠐⠣⠭⠐⠜ ⠯ ⠮
⠐⠏⠊⠁⠇ ⠙⠻⠊⠧⠁⠞⠊⠧⠑ ⠊⠎ ⠰⠷⠨⠙⠽⠨⠌⠨⠙⠭⠰⠾⠲

OR

⠠⠊⠋ ⠰⠽ ⠐⠶ ⠋⠐⠣⠭⠐⠜ ⠮⠝ ⠮ ⠙⠻⠊⠧⠁⠞⠊⠧⠑
⠊⠎ ⠰⠰⠷⠙⠽⠨⠌⠙⠭⠾ ⠕⠗ ⠋⠶⠐⠣⠭⠐⠜ ⠯ ⠮
⠐⠏⠊⠁⠇ ⠙⠻⠊⠧⠁⠞⠊⠧⠑ ⠊⠎ ⠰⠰⠷⠨⠙⠽⠨⠌⠨⠙⠭⠾⠲

AND

$$\frac{dy}{dx} = 0$$

⠰⠰⠷⠙⠽⠨⠌⠙⠭⠾ ⠐⠶ ⠼⠚

OR

⠰⠷⠙⠽⠨⠌⠙⠭⠰⠾ ⠐⠶ ⠼⠚

$$\frac{d^2v}{dt^2}$$

⠰⠰⠷⠙⠔⠼⠃⠧⠨⠌⠙⠞⠔⠼⠃⠾

OR

⠰⠷⠙⠰⠔⠼⠃⠧⠨⠌⠙⠞⠔⠼⠃⠾

$f'(x) = 0$

⠋⠶⠐⠣⠭⠐⠜ ⠐⠶ ⠼⠚

$f''(x) = 0$

⠋⠶⠶⠐⠣⠭⠐⠜ ⠐⠶ ⠼⠚

$\frac{d}{dx}(kx) = k$

⠰⠰⠰⠷⠙⠨⠌⠙⠭⠾⠐⠣⠅⠭⠐⠜ ⠐⠶ ⠅⠰⠄

OR

⠰⠰⠷⠙⠨⠌⠙⠭⠾⠐⠣⠅⠭⠐⠜ ⠐⠶ ⠰⠅

OR

⠰⠷⠙⠨⠌⠙⠭⠰⠾⠐⠣⠅⠭⠐⠜ ⠐⠶ ⠰⠅

Remember:

As referred to in the General Note in Lesson 1, there is often a choice of Grade 1 indicator, with any of the options of Grade 1 mode (symbol, word, or passage) being equally correct.

Decisions about option selection are generally associated with user and transcriber preferences, or local production formatting guidelines, and include consideration for simplicity or functionality.

Therefore, for the purpose of the exercises in the UEB Mathematics Online, use the following criteria for implementing Grade 1 mode:

- Use the Grade 1 **symbol** indicator when there is only one symbol in the sequence requiring a Grade 1 indicator.
- Use the Grade 1 **word** indicator when there are two or more symbols in the sequence requiring Grade 1 mode except in a context whereby any literary elements will be affected and as a consequence will also be uncontracted.
- Use Grade 1 **passage** indicator (with Grade 1 terminator) when a comparison indicator or a space is used in the sequence and Grade 1 indicators are required on both sides of the equation. Remember however, the impact of Grade 1 mode on any literary elements.
- As stated in 10.6.5 of "*The Rules of Unified English Braille"* the lower groupsign for "gg" is only used when the letters it represents are both preceded and followed by a letter, a contraction, a modified letter or a ligatured letter. Therefore, the sign for differentiation will generally not require a grade 1 indicator as it will not be misread.

Hints:

- The numeric indicator also sets Grade 1 mode for the remainder of the symbols-sequence.
- Insert a space if a function name is **preceded** directly by a lower or upper-case Latin letter, with no intervening braille indicators or brackets.
- Insert a space if a function name is **followed** directly by a lower-case Latin letter, with no intervening braille indicators or brackets.
- If the braille representation for a print sequence does not fit on the line, then a dot 5 (⠐) continuation indicator placed at a logical place immediately following the last character may be required to show the braille is continuing to the next line. Usually the preferred place to break is before a comparison sign, an operation sign, or a fraction line indicator. For the purposes of this training program take the remainder of the sequence to the margin as it is recognised there may be various local formatting guidelines.

Exercise 6

1. $\frac{dy}{dx} > 0$

2. $\frac{dy}{dx} f^{'}(x)$

3. $\frac{dy}{dx} f^{'}(x) e^{f(x)}$

4. $\frac{d}{dx}(x^2 - 2x)$

5. $\frac{dy}{dx} = \frac{2}{\sqrt{1-x^2}}$

6. $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

7. $\frac{d}{dx}(kx^n) = knx^{n-1}$

8. $\frac{dy}{dx} = 4x^3 - 9x^2 + 7$

9. $\frac{dy}{dx} = f'(x) \cos f(x)$

10. $\frac{d}{dx}[f(x)]^2 =$

Review Exercise 6

1. $\log_b x = \frac{\log_a x}{\log_a b}$

2. $\frac{\delta y}{\delta x} = \frac{\delta y}{\delta u} \times \frac{\delta u}{\delta x}$

3. $\frac{dy}{dx} = nx^{nx-1}$

4. $\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}$

5. $\frac{dy}{dx} = \frac{f'(x)}{f(x)}$

6. $\frac{dy}{dx} = 5x^4 - 9$

7. $\left(\frac{de^5 f}{def^4}\right)^0$

8. $\lim_{\theta \to 0} \frac{tan\frac{\theta}{3}}{\theta}$

9. $\lim_{x \to \infty} \frac{3\sqrt{x}}{\sqrt{x} - 1}$

10. $\lim_{x \to -1} \frac{x^2 + 4x + 1}{x^2 - 1}$

Lesson 7 - Calculus: Integration

As Calculus is a defined topic in senior secondary level mathematics it has been introduced specifically into the lessons for this training program. The same rules and principles of braille transcription should be observed that have been introduced previously.

There are two main branches of Calculus: Differentiation and Integration. The topic of Differentiation was introduced in Lesson 6.

Integration as presented in this Lesson is the inverse of differentiation and uses information about rates of change to go back and examine the original variables. Integration can also be used to find areas of curved objects.

This training program will only be introducing the standard integral sign ($\int$) at the level of Mathematics being presented in this training program and is shown in braille as (⠮).

Notes:

- The spacing of the integral sign in print ($\int$) may be unclear or inconsistent. In braille it is best to have the integral sign (⠮) unspaced from the function and treat its limits as subscripts and superscripts. The *dx* at the end means "integrate with respect to *x*" and can also be written unspaced.
- While both of the examples below are correct, the first example in each using Grade 1 word mode is the preferred option due to requiring more than one (1) Grade 1 symbol indicator in the sequence.

  Examples:

  $$\int \frac{1}{\sqrt{a^2 - \ x^2}}\ dx$$

  ⠰⠰⠮⠷⠼⠁⠨⠌⠩⠁⠔⠼⠃⠐⠤⠭⠔⠼⠃⠬⠾⠙⠭ preferred

  OR

  ⠰⠮⠰⠷⠼⠁⠨⠌⠩⠁⠔⠼⠃⠐⠤⠭⠔⠼⠃⠬⠾⠙⠭

  $$\int_2^3 (2x - 1)dx$$

  ⠰⠰⠮⠢⠼⠃⠔⠼⠉⠐⠣⠼⠃⠭⠐⠤⠼⠁⠐⠜⠙⠭ preferred

  OR

  ⠰⠮⠰⠢⠼⠃⠔⠼⠉⠐⠣⠼⠃⠭⠐⠤⠼⠁⠐⠜⠙⠭

Remember:

As referred to in the General Note in Lesson 1, there is often a choice of Grade 1 indicator, with any of the options of Grade 1 mode (symbol, word, or passage) being equally correct.

Decisions about option selection are generally associated with user and transcriber preferences, or local production formatting guidelines, and include consideration for simplicity or functionality.

Therefore, for the purpose of the exercises in UEB Mathematics Online, use the following criteria for implementing Grade 1 mode:

- Use the Grade 1 **symbol** indicator when there is only one symbol in the sequence requiring a Grade 1 indicator.
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- Use Grade 1 **passage** indicator (with Grade 1 terminator) when a comparison indicator or a space is used in the sequence and Grade 1 indicators are required on both sides of the equation. Remember however, the impact of Grade 1 mode on any literary elements.
- As stated in 10.6.5 of "*The Rules of Unified English Braille"* the lower groupsign for "gg" is only used when the letters it represents are both preceded and followed by a letter, a contraction, a modified letter or a ligatured letter. Therefore, the sign for differentiation will generally not require a grade 1 indicator as it will not be misread.

Hints:

- The numeric indicator also sets Grade 1 mode for the remainder of the symbols-sequence.
- Insert a space if a function name is **preceded** directly by a lower or upper-case Latin letter, with no intervening braille indicators or brackets.
- Insert a space if a function name is **followed** directly by a lower-case Latin letter, with no intervening braille indicators or brackets.
- Signs of operation and comparison are unspaced when they appear in an expression which is not on the base line such as directly above or directly below the character.
- If the braille representation for a print sequence does not fit on the line, then a dot 5 (⠐) continuation indicator placed at a logical place immediately following the last character may be required to show the braille is continuing to the next line. Usually the preferred place to break is before a comparison sign, an operation sign, or a fraction line indicator. For the purposes of this training program take the remainder of the sequence to the margin, as it is recognised there may be various local formatting guidelines.

Exercise 7

1. $\int x^2\, dx$

2. $\int_0^2 \frac{3}{4+x^2}\, dx$

3. $\int 2x^4\ dx$

4. $\int_3^{\pi} \cos\left(\frac{x}{2}+\ \pi\right)dx$

5. $\int_0^5 \frac{dx}{x+3}$

6. $\int_a^b f(x)dx$

7. $\int \sin^2\ x\, dx$

8. $\int e^{ax+b}\, dx$

9. $\int \tan x dx = \ln\cos x + c$

10. $\pi \int_0^{\frac{\pi}{4}} sec^2\ x\ dx$

Review Exercise 7

1. $f''(x) = 6x - 8$

2. $\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$

3. $\frac{d^2x}{dt^2}$

4. $P(E') = 1 - p(E)$

5. $\frac{dy}{dx} = F'(u)\frac{du}{dx}$

6. $\int sec^2\,(ax+b)dx$

7. $\lim\limits_{\theta \to 0} \frac{sin\theta}{\theta} = 1$

8. $\int \cos{(ax+b)}\, dx$

9. $\int_1^2 \frac{x^3 - x^2 + 5x + 3}{x}\ dx$

10. $\int (ax+b)^n$

Lesson 8 - Vertical Bars

One of the underlying principles of UEB is that each print symbol should have one and only one braille equivalent. For example, the vertical bar is used in print to represent absolute value, conditional probability, and the words “such as”. The same braille symbol should be used in all these cases.

Bars are usually unspaced in print from the items they enclose and the same should be done in braille.

The absolute value of a number, for example, is symbolised in print by two vertical lines | | often referred to as bars and shown in braille as

opening ⠸⠳ and closing ⠸⠳.

Examples:

$|x-2|$

⠸⠳⠭⠐⠤⠼⠃⠸⠳

$|x|^3 = x^3$

⠰⠰⠰⠸⠳⠭⠸⠳⠔⠼⠉ ⠐⠶ ⠭⠔⠼⠉⠰⠄

OR

⠸⠳⠭⠸⠳⠰⠔⠼⠉ ⠐⠶ ⠭⠰⠔⠼⠉

$\{(x,\ y)\ |\ x+y\ =\ 6\}$

⠰⠰⠰⠸⠣⠐⠣⠭⠂ ⠽⠐⠜ ⠸⠳ ⠭⠐⠖⠽ ⠐⠶ ⠼⠋⠸⠜⠰⠄

OR

⠸⠣⠐⠣⠰⠭⠂ ⠰⠽⠐⠜ ⠸⠳ ⠭⠐⠖⠽ ⠐⠶ ⠼⠋⠸⠜

$y = |2x-\ 1|$

⠰⠽ ⠐⠶ ⠸⠳⠼⠃⠭⠐⠤⠼⠁⠸⠳

$$\frac{|x-3|}{3-x} = x$$

⠰⠰⠰⠷⠸⠳⠭⠐⠤⠼⠉⠸⠳⠨⠌⠼⠉⠐⠤⠭⠾ ⠐⠶ ⠭⠰⠄

OR

⠰⠷⠸⠳⠭⠐⠤⠼⠉⠸⠳⠨⠌⠼⠉⠐⠤⠭⠾ ⠐⠶ ⠰⠭

$\left|\overrightarrow{AB}\right|$

⠰⠰⠸⠳⠣⠠⠠⠁⠃⠜⠘⠱⠸⠳

Remember:

As referred to in the General Note in Lesson 1, the options of Grade 1 mode (symbol, word, or passage) are equally correct in mathematical contexts.

Decisions about option selection are generally associated with user and transcriber preferences, or local production formatting guidelines, and include consideration for simplicity or functionality.

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- Use Grade 1 **passage** indicator (with Grade 1 terminator) when a comparison indicator or a space is used in the sequence and Grade 1 indicators are required on both sides of the equation. Remember however, the impact of Grade 1 mode on any literary elements.
- As stated in 10.6.5 of "*The Rules of Unified English Braille"* the lower groupsign for "gg" is only used when the letters it represents are both preceded and followed by a letter, a contraction, a modified letter or a ligatured letter. Therefore, the sign for differentiation will generally not require a grade 1 indicator as it will not be misread.

Hints:

- The numeric indicator also sets Grade 1 mode for the remainder of the symbols-sequence.
- Signs of operation and comparison are unspaced when they appear in an expression which is not on the base line such as directly above or directly below the character.

Exercise 8

1. $|y-4|$
2. $\frac{|x|}{x^2} = 1$
3. $|x| \geq 0$
4. $\sqrt{a^2} = |a|$
5. $\frac{|3x-2|}{3x-2} = 1$
6. $|3y-1| + |2y+3| > 5$
7. $y^2 = 2\sin|x|$
8. $|z-2| + |z+1| = 7$
9. $|3x+2| = 2x-3$
10. $\left|\overrightarrow{O_1 P}\right|$

Review Exercise 8

1. $A \approx \frac{h}{2}(d_f + d_l)$
2. $(x-h)^2 = \pm 4a(y-k)$
3. $y = e^{f(x)}$
4. $\frac{p^{\frac{1}{2}} \times p^{\frac{1}{2}}}{p^2}$
5. $S_n = \frac{a(r^n-1)}{r-1}$
6. $x^{\frac{1}{n}} = \sqrt[n]{x}$
7. $\frac{1}{\sqrt[4]{(7-x)^2}}$

8. $\left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

9. $f'(x)n[f(x)]^{n-1}$

10. $\lim_{h \to 0} \frac{2xh - h^2 \ -5h}{h}$

Lesson 9 - Review Test

Congratulations on reaching this Review Test. The content of each lesson has built upon preceding lessons, with the overall structure of the training manual designed to reinforce several foundational principles, including the following:

1. The numeric indicator sets numeric mode **and** Grade 1 mode for the remainder of the symbols-sequence unless terminated for any reason.
2. A braille symbol may have both an uncontracted (Grade 1) meaning and a contracted (Grade 2) meaning. A Grade 1 indicator is used to set Grade 1 mode when the Grade 1 meaning of a symbol could be misread as a contraction meaning or a numeric meaning.
3. When is a fraction considered a "simple fraction"? Note, if the sequence does not satisfy the definition of a simple fraction, then it must be treated as a general fraction.
4. The rules associated with "the next item" should be considered and are particularly relevant in the use of level change indicators.

Remember:

As referred to in the General Note in Lesson 1 in this UEB Training Manual: Extension Mathematics and throughout the lessons, the choice of options for Grade 1 mode (symbol, word, or passage) may be equally correct in mathematical contexts.

Decisions about option selection are generally associated with user and transcriber preferences, or local production formatting guidelines, and include consideration for simplicity or functionality.

Therefore, for the purpose of the exercises in UEB mathematics Online, use the following criteria for implementing Grade 1 mode:

- Use the Grade 1 **symbol** indicator when there is only one symbol in the sequence requiring a Grade 1 indicator.
- Use the Grade 1 **word** indicator when there are two or more symbols in the sequence requiring Grade 1 mode except in a context whereby any literary elements will be affected and as a consequence will also be uncontracted.
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- As stated in 10.6.5 of "*The Rules of Unified English Braille*" the lower groupsign for "gg" is only used when the letters it represents are both preceded and followed by a letter, a contraction, a modified letter or a ligatured letter. Therefore, the sign for differentiation will generally not require a grade 1 indicator as it will not be misread.

Hints:

- The numeric indicator also sets Grade 1 mode for the remainder of the symbols-sequence.
- Signs of operation and comparison are unspaced when they appear in an expression which is not on the base line such as directly above or directly below the character.
- If the braille representation for a print sequence does not fit on the line, then a dot 5 (⠐) continuation indicator placed at a logical place immediately following the last character may be required to show the braille is continuing to the next line. Usually the preferred place to break is before a comparison sign, an operation sign, or a fraction line indicator. For the purposes of this training program take the remainder of the sequence to the margin, as it is recognised there may be various local formatting guidelines.

Review Test

1. $8.754\dot{7}5\dot{9}$
2. $T_n = a \ + (n-1)$
3. $\bar{y} = -10$
4. $\overline{AB}$
5. $\ln|f(x)| + c$
6. $\bar{x} = \frac{\Sigma x}{n}$
7. $|5x - 3| \geq 2$
8. $\frac{(2x^7)^3 y^2}{x^{10} y}$
9. $[2(x + y)] \div 4 < 10$
10. $|-8| = |8| = 8$
11. $\cos C = \frac{a^2+b^2-c^2}{2a}$
12. $A = \int_{-1}^{2} (2 + x)\, dx$
13. $\int_b^c f(x) dx$
14. $\sin(\pi - \frac{\pi}{8})$
15. $\frac{n!}{r!(n-r)!}$
16. $\theta = n\pi \ + (-1)^n \sin^{-1} a$
17. $\lim\limits_{x \to 0} f(x)$
18. $f_1(x) + f_2(x)$
19. $S_n = \frac{n}{2}[2a \ + \ (a - l)d]$

20. $f''(x) = k(b^2 - x^2)$

Lesson 10 - Extension Test

Congratulations on reaching this Extension Test. As noted in Lesson 9, the content of each lesson has built upon preceding lessons, with the overall structure of the training manual designed to reinforce several foundational principles, including the following:

1. The numeric indicator sets numeric mode **and** Grade 1 mode for the remainder of the symbols-sequence unless terminated for any reason.
2. A braille symbol may have both an uncontracted (Grade 1) meaning and a contracted (Grade 2) meaning. A Grade 1 indicator is used to set Grade 1 mode when the Grade 1 meaning of a symbol could be misread as a contraction meaning or a numeric meaning.
3. When is a fraction considered a “simple fraction”? Note: If the sequence does not satisfy the definition of a simple fraction, then it must be treated as a general fraction.
4. The rules associated with “the next item” should be considered and are particularly relevant in the use of level change indicators.

Remember:

As referred to in the General Note in Lesson 1 in this UEB Training Manual: Extension Mathematics and throughout the previous lessons, the choice of options for Grade 1 mode (symbol, word, or passage) may be equally correct in mathematical contexts.

Decisions about option selection are generally associated with user and transcriber preferences, or local production formatting guidelines, and include consideration for simplicity or functionality.

Therefore, for the purpose of the Extension Test in this UEB Training Manual, use the following criteria for implementing Grade 1 mode:

- Use the Grade 1 **symbol** indicator when there is only one symbol in the sequence requiring a Grade 1 indicator.
- Use the Grade 1 **word** indicator when there are two or more symbols in the sequence requiring Grade 1 mode except in a context whereby any literary elements will be affected and as a consequence will also be uncontracted.
- Use Grade 1 **passage** indicator (with Grade 1 terminator) when a comparison indicator or a space is used in the sequence and Grade 1 indicators are required on both sides of the equation. Remember however, the impact of Grade 1 mode on any literary elements.
- As stated in 10.6.5 of “*The Rules of Unified English Braille”* the lower groupsign for “gg” is only used when the letters it represents are both preceded and followed by a letter, a contraction, a modified letter or a ligatured letter. Therefore, the sign for differentiation will generally not require a grade 1 indicator as it will not be misread.

Hints:

- The numeric indicator also sets Grade 1 mode for the remainder of the symbols-sequence.
- Insert a space if a function name is **preceded** directly by a lower or upper-case Latin letter, with no intervening braille indicators or brackets.
- Insert a space if a function name is **followed** directly by a lower-case Latin letter, with no intervening braille indicators or brackets.
- Signs of operation and comparison are unspaced when they appear in an expression which is not on the base line such as directly above or directly below the character.
- If the braille representation for a print sequence does not fit on the line, then a dot 5 (⠐) continuation indicator placed at a logical place immediately following the last character may be required to show the braille is continuing to the next line. Usually the preferred place to break is before a comparison sign, an operation sign, or a fraction line indicator. For the purposes of this training program take the remainder of the sequence to the margin as it is recognised there may be various local formatting guidelines.

Extension Test

1. $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

2. $\int x^2 dx$

3. $\int \frac{\cos x}{\sin x}\, dx$

4. $\int_2^3 \frac{dx}{x}$

5. $d = \sqrt{(x_2 - x_1)}$

6. $\operatorname{cosec} A = \frac{1}{\sin A},\ sin\, A \neq 0$

7. $(Re(\alpha))^2 + (Re(\beta))^2 = 1$

8. $\int \frac{6x^2}{x^3}$

9. $d = \frac{|ax_1|}{\sqrt{a^2}}$

10. $\frac{d}{dx}\left(\frac{x^2}{x-1}\right)$

11. $\bar{x} = \frac{\sum x}{n}$

12. $\sum_2^4 |3p - 2|$

13. $\ddot{x} = -n^2(x - c)$

14. $y = \tan^{-1} f(x)$

15. $A_n = P\left(1 + \frac{r}{100}\right)^n$

16. $(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n})$

17. $\max_{0 \leq x \leq 1} xe^{-x^2}$

18. $\sum_{i=1}^{n} x_1 p_1$

19. $\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$

20. $\int \frac{f'(x)}{f(x)}dx$

References

Howse, J. (2018a). Unified English Braille Maths K – 6. Workshop presented at RIDBC Renwick Centre, North Rocks, NSW. Unpublished document.

Howse, J. (2018b). Unified English Braille Maths 7 – 12 Workshop presented at RIDBC Renwick Centre, North Rocks, NSW. Unpublished document.

Howse, J. (2019a). Unified English Braille training manual: Introductory mathematics (Revision 5). https://uebonline.org/resources/ueb-mathematics-resources/

Howse, J. (2019b). Unified English Braille training manual: Advanced Mathematics (Revision 3). https://uebonline.org/resources/ueb-mathematics-resources/

Howse, J. Riessen, K. and Holloway, L. (Eds.). (2016). Unified English Braille: Australian Training Manual. http://brailleaustralia.org/unified-english-braille/unified-english-braille-australian-training-manual-2013/

International Council on English Braille. (2014). Unified English Braille Guidelines for Technical Material. http://www.iceb.org/guidelines_for_technical_material_2014.pdf

Simpson, C. (Ed.). (2013).

The Rules of Unified English Braille (2013) (2nd ed.). International Council on English Braille (ICEB). http://iceb.org/Rules of Unified English Braille 2013.pdf.