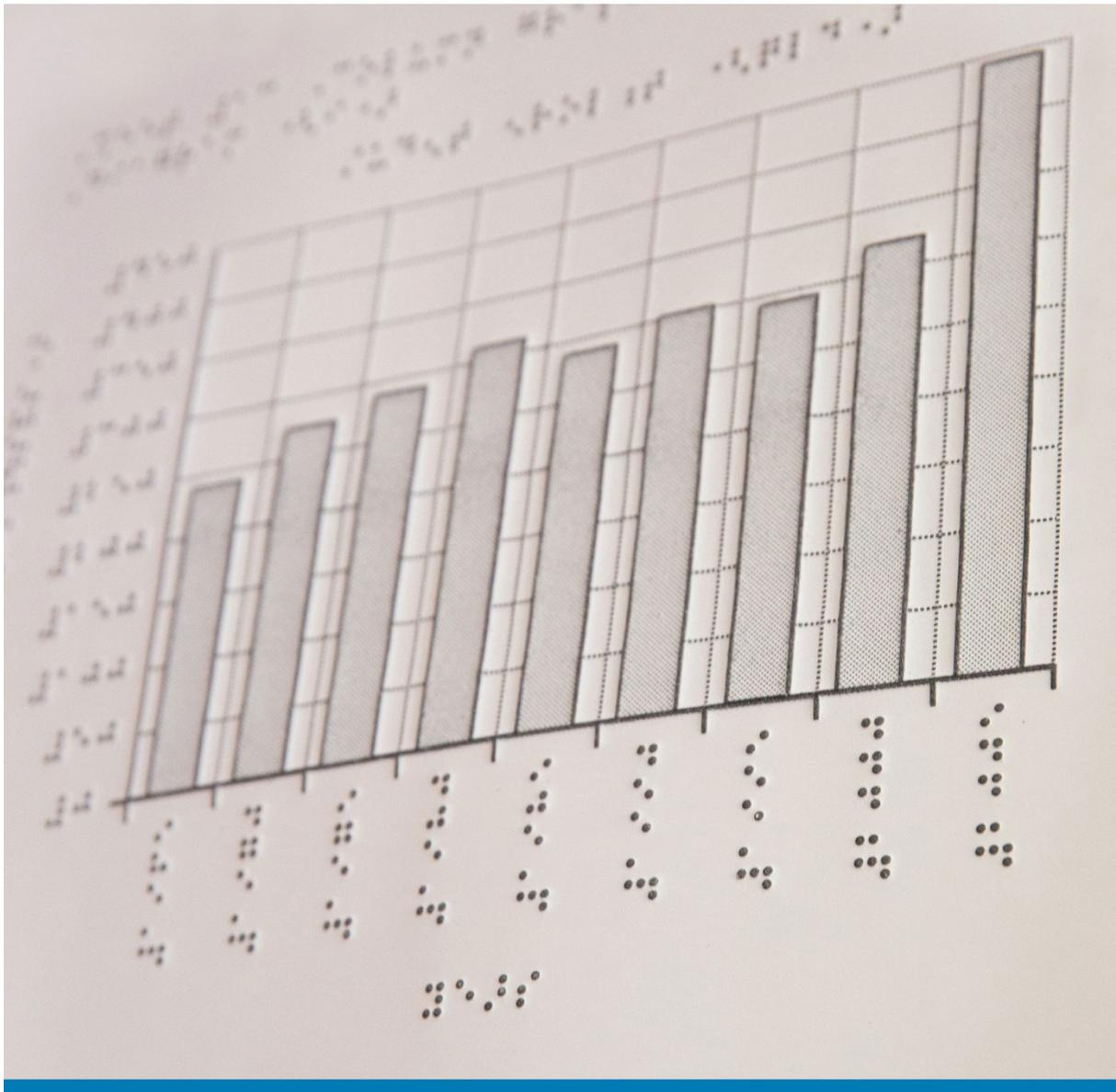


Unified English Braille Training Manual: **Advanced Mathematics**

Josie Howse



Unified English Braille Training Manual: Advanced Mathematics

Revision 3

© 2019 North Rocks Press

Royal Institute for Deaf and Blind Children

361-365 North Rocks Road

North Rocks NSW 2151

Australia

<https://www.ridbc.org.au/>

This work is licensed under the Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International Licence.

- Attribution: You must give appropriate credit to the author
- Non-Commercial: You must not use the material for commercial purposes
- No derivatives: If you remix, transform or build upon the material, you may not distribute the modified material.

To view a copy of this licence, visit <https://creativecommons.org/licenses/by-nc-nd/4.0/> or send a letter to Creative Commons, 171 Second Street, Suite 300, San Francisco, California 94105 USA



ISBN 978-0-949050-06-9



A catalogue record for this book is available from the National Library of Australia

The RIDBC Renwick Centre for Research and Professional Education is administered by the Royal Institute for Deaf and Blind Children. The Centre is committed to the provision of high quality teaching and learning opportunities for professionals and families supporting people with vision or hearing impairment, in addition to research and related community service in these same areas.

Table of Contents

Foreword	5
Contributors	7
Advanced Mathematics	8
Lesson 1: Grade 1 Mode (Symbol Indicator) and Algebra	8
Grade 1 Mode	8
Grade 1 Symbol Indicator	8
Numeric Indicators Set Grade 1 Mode	9
Standing Alone Rule	10
Algebra	11
Exercise 1	13
Extra Exercise 1	13
Lesson 2: Grade 1 Mode (Word and Passage) and Fractions (continued)	14
Grade 1 Mode	14
Grade 1 Word Indicator	14
Grade 1 Passage Indicator and Grade 1 Passage Terminator	15
Grade 1 Passage Terminator	15
Grade 1 Passage Indicator and Terminator on a line of their own	15
Fractions	16
Simple numeric fraction definition (Revision)	17
General Fractions	18
Exercise 2	21
Review Exercise 2	21
Lesson 3: Operation and Comparison Signs (continued)	22
Operation Signs	22
Comparison Signs	22
Exercise 3	25
Review Exercise 3	25
Lesson 4: Indices	26
Superscripts and Subscripts	26
Level Change Indicator	26
Braille Grouping Indicator	26

Definition of an Item	27
Exercise 4	31
Review Exercise 4	31
Lesson 5: Roots and other Radicals.....	32
Square Roots	32
Radical Index (Cube Roots etc)	32
Square Root sign on its own with no vinculum	33
Exercise 5.....	35
Review Exercise 5.....	35
Lesson 6: Shape Indicators (continued) and Miscellaneous Symbols	36
Shape Indicators	36
Transcriber defined shapes	36
Use of the shape termination indicator	37
Miscellaneous Symbols (continued)	37
Exercise 6.....	41
Review Exercise 6.....	41
Lesson 7: Functions	42
Trigonometric functions.....	42
Logarithmic functions	43
Exercise 7.....	44
Review Exercise 7.....	44
Lesson 8: Greek Letters	45
Exercise 8.....	46
Review Exercise 8.....	46
Lesson 9: Review Test	47
Review Test.....	48
Lesson 10: Advanced Test.....	49
Advanced Test	50
References	51

Foreword

If you are reading this Foreword, it is most likely that you have successfully completed the online Unified English Braille (UEB) Introductory Mathematics training program. Please accept my congratulations for your success to date!

The **purpose** of this UEB Advanced Mathematics training program is to provide instruction in the reading and writing of secondary-level mathematical symbols in Unified English Braille. Mathematics is widely considered a core learning area and an essential requirement for the study of the STEM subjects of Science, Technology and Engineering. Mathematics teachers generally possess specialised mathematical knowledge and skills that enable them to effectively teach secondary students with a diverse range of abilities and attributes. However, students who use braille to access and communicate information require instruction from teachers who understand the braille code and are able to effectively modify print-based information into tactile form.

The **target audience** for UEB Advanced Mathematics includes mathematics teachers, teachers of braille, parents and caregivers, allied health professionals, education administrators and policymakers.

The **aims** of the UEB Advanced Mathematics training program are:

- To promote the acquisition of knowledge of Unified English Braille as it is applied to the continuum of those mathematical symbols that are typically taught during the secondary years of schooling; and
- To raise awareness of the enormous potential of braille knowledge and skills in enabling students with vision impairment to effectively access and engage with mathematics content and to communicate their mathematical understanding in a broad range of contexts.

The **instructional content** is presented as a series of lessons that address specific topics in secondary education. The lessons include practice and review exercises involving print to braille transcription. The content of each lesson builds on prior content, enabling the progressive development and consolidation of braille knowledge.

Please **note** that an additional UEB Online training program called UEB Extension Mathematics addresses mathematics content that is encountered during the senior years of secondary mathematics – see <https://uebonline.org>.

The **recommended UEB Online study sequence** is: (i) completion of UEB Literacy modules 1 and 2; followed by (ii) UEB Introductory Mathematics; and then (iii) UEB Advanced Mathematics and (iv) UEB Extension Mathematics. This step-by-step sequence is recommended because Unified English Braille is a single code system that encompasses the braille symbols for literary and technical information. It is

important, therefore, to be knowledgeable of the UEB symbols used in literary contexts, as these symbols are used for literary content of mathematical information.

On behalf of the RIDBC Renwick Centre, Royal Institute for Deaf and Blind Children, I extend my sincere thanks to those organisations and individuals who have contributed to developing the UEB Online mathematics training programs and supporting materials – see <https://uebonline.org>. This includes thanks to my Project Team colleagues, Josie Howse (content author), Craig Cashmore of Peppacode Pty Ltd (online program developer); and RIDBC's Lena Karam, Sonali Marathe and Tarna Cosgrove who have assured the accuracy and accessibility of the information presented. Sincere appreciation is also extended to the Duchon Family Foundation, the JLDJS Foundation, Sibley Endowment, the Skipper-Jacobs Charitable Trust and the Thomas Hare Investments Trust. Without their financial support, the UEB Online Mathematics training programs would not have been possible.

We hope that this contribution to the disability field will provide professionals and parents with the required knowledge to support and encourage secondary school students with vision impairment to actively and confidently engage with the exciting world of mathematics.

Frances Gentle, AO PhD

UEB Online Project Team Leader and Conjoint Lecturer,
RIDBC Renwick Centre, Royal Institute for Deaf and Blind Children

Contributors

Author: Josie Howse, PSM

M.Spec. Ed. (Sensory Disability) – University of Newcastle,
BA (Ed.) – Macquarie University

Josie Howse is an Adjunct Research Fellow with the RIDBC Renwick Centre and former Manager of the Braille and Large Print Services, NSW Department of Education. The NSW Department of Education team provides all texts and examinations in braille, large print and e-text to students with vision impairment in the government sector and is the largest producer of alternative format student textbooks and examinations in Australia.

Josie has been working in the field of vision impairment for more than 40 years. She has held a number of executive positions at national and state levels and has extensive experience in braille code development at an international level. Josie was the editor of the 2006 Unified English Braille Primer: Australian Edition, and co-editor of the 2016 Unified English Braille: Australian Training Manual. Josie was awarded the Public Service Medal (PSM) in the Queen's Birthday Honours list in 2007, has been listed annually in Who's Who of Australian Women since 2008, and is the recipient of a Lifetime Achievement Award from the Round Table on Information Access for People with Print Disabilities in 2012.

UEB Online Developer: Craig Cashmore

B. Eng. (Hons) – UTS

Craig holds an Engineering Degree in Telecommunications and has worked in the software development industry for over 30 years, holding senior software design, software architecture and technical management positions in companies including Jtec, Ericsson and LongReach Networks.

More recently Craig founded Peppacode, a web and app development business focused on 'out-of-the-ordinary' strategic web and software development for small business, start-ups and educational institutions.

Some of Craig's achievements at Peppacode include the successful launch of UEB Online for RIDBC and a vehicle tracking and management system for a bus operator. Craig continues to work on new and innovative projects using modern web technologies.

Advanced Mathematics

Lesson 1: Grade 1 Mode (Symbol Indicator) and Algebra

Grade 1 Mode

The rules for the use of Grade 1 mode in a literary context will also apply in a mathematical context. Please revise the information presented in Lesson 3 of the *Unified English Braille Training Manual: Introductory Mathematics*.

A braille symbol may have both an uncontracted (Grade 1) meaning and a contracted (Grade 2) meaning. A Grade 1 indicator is used to set Grade 1 mode when the Grade 1 meaning of a symbol could be misread as a contraction meaning or a numeric meaning. The extent of Grade 1 mode is determined by the Grade 1 indicator in use.

⠠ Grade 1 Symbol indicator (dots 56)

The following Grade 1 mode indicators will be introduced more fully in Lesson 2 of this Training Manual.

⠠⠠ Grade 1 Word Indicator

⠠⠠⠠ Grade 1 Passage Indicator

⠠⠠⠠ Grade 1 Passage Terminator

⠠⠠⠠⠠⠠ Grade 1 Passage Indicator on a line of its own

⠠⠠⠠⠠⠠ Grade 1 Passage Terminator on a line of its own

Note:

Grade 1 indicators will not be needed for simple arithmetic problems involving number, operation signs, numerical fractions and mixed numbers.

Grade 1 Symbol Indicator

- A Grade 1 symbol indicator (⠠) sets Grade 1 mode for only the next symbol.

Example:

Seat 10a

⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠

- A Grade 1 symbol indicator is NOT required before the letters a, i and o, because they do not have a contraction meaning when they “stand alone”.

Examples:

(c

⠠⠠⠠⠠⠠⠠

(C

⠠⠠⠠⠠⠠⠠⠠

"can

⠠⠠⠠

"p's and q's"

⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

General Note

Decisions often need to be made in a mathematical context about whether to use:

- a Grade 1 symbol indicator or
- a Grade 1 word indicator or
- a Grade 1 passage indicator with a Grade 1 terminator or
- a Grade 1 passage indicator on a line of its own with a Grade 1 passage terminator on a line of its own.

Often there is a choice about Grade 1 indicators in mathematical contexts, with any of the options (above) being equally correct. Decisions about option selection are generally associated with user and transcriber preferences, including consideration for simplicity or functionality.

Algebra

- Algebra is the part of mathematics in which letters and other general symbols are used to represent numbers and quantities in formulae and equations.
- Simple algebraic equations which include letters may need Grade 1 symbol indicators where the letters stand alone, or the letters a-j immediately follow the numbers and the letters themselves may be misread as numbers.
- The ratio sign ($\frac{\cdot}{\cdot}$) has a Grade 2 (contracted) meaning, so unless you are already in Grade 1 mode, a Grade 1 symbol indicator will be required before the ratio sign.
- Remember that the numeric indicator also sets Grade 1 mode for the next symbols-sequence. When Grade 1 mode is set by a numeric indicator, it is terminated by a space, a hyphen, a dash or a Grade 1 terminator.

Notes:

1. The presentation of algebraic expressions in print is often shown in italics. This is generally ignored in braille.
2. If the braille representation for a print sequence in the following exercises does not fit on the line, then the first preference would be to break:
 - before comparison signs,
 - before operation signs, or
 - before a mathematical unit such as
 - fractions
 - functions
 - radicals
 - items with modifiers such as superscripts or bars
 - shapes or arrows
 - anything enclosed in print or braille grouping symbols.

Usually the best place to break is before a comparison sign or an operation sign.

Examples:

$$x = y + 5c$$

⠠⠭ ⠦ ⠠⠽ ⠦ ⠠⠑ ⠦ ⠠⠎

Seat 6a

⠠⠑⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠

The ratio of x : y

⠠⠭ ⠦ ⠠⠽ ⠦ ⠠⠑ ⠦ ⠠⠎

The ratio 5: 10

⠠⠑ ⠦ ⠠⠠⠠⠠⠠ ⠦ ⠠⠠⠠⠠⠠

Exercise 1

1. $a + b = c$
2. $x + y = z$
3. $2x = y$
4. $am - an$
5. x -axis
6. X -axis
7. The product of a and b , (ab) , gives the area.
8. The ratio $p:q$
9. $Ab = Xb + Yb$
10. $4r \div 2r =$

Extra Exercise 1

1. Expand $(a + 2)(a + 4)$
2. Expand and simplify $(3a + 5)(a + 1)$
3. $(q + 4)(q - 2)$
4. $2x(3x - 1) + 3(3x - 1)$
5. $ab + cd = gd$
6. $(m + 3)(m + 3) = m(m + 3) + 3(m + 3)$
7. $21a \div 7a = 3$
8. $mn + yz =$
9. $7b + 5b =$
10. $(t - 4)(t - 7)$

Lesson 2: Grade 1 Mode (Word and Passage) and Fractions (continued)

Grade 1 Mode

The rules for the use of Grade 1 mode in a literary context will also apply in a mathematical context. Please revise the information presented in Lesson 3 of the *Unified English Braille Training Manual: Introductory Mathematics* and Lesson 1 of this Training Manual.

A braille symbol may have both an uncontracted (Grade 1) meaning and a contracted (Grade 2) meaning. A Grade 1 indicator is used to set Grade 1 mode when the Grade 1 meaning of a symbol could be misread as a contraction meaning or a numeric meaning. The extent of Grade 1 mode is determined by the Grade 1 indicator in use.

⠠	Grade 1 Symbol Indicator (dots 56)
⠠⠠	Grade 1 Word Indicator
⠠⠠⠠	Grade 1 Passage Indicator
⠠⠠⠠⠠	Grade 1 Passage Terminator
⠠⠠⠠⠠⠠	Grade 1 Passage Indicator on a line of its own
⠠⠠⠠⠠⠠⠠	Grade 1 Passage Terminator on a line of its own

Note:

Grade 1 indicators will not be needed for simple arithmetic problems involving number, operation signs, numerical fractions and mixed numbers.

Grade 1 Word Indicator

- The Grade 1 word indicator (⠠⠠) sets Grade 1 mode for the following sequence of symbols or the remainder of the current symbol sequence.
- The effect of a Grade 1 word indicator is terminated by a space or a Grade 1 terminator.
- Remember that the numeric indicator (⠠) also sets Grade 1 mode for the next symbols-sequence.
- Complex algebraic expressions that do **not** include a comparison sign are best shown using Grade 1 word mode.
- More examples of the use of Grade 1 word mode in a mathematical context will be presented later in this manual.

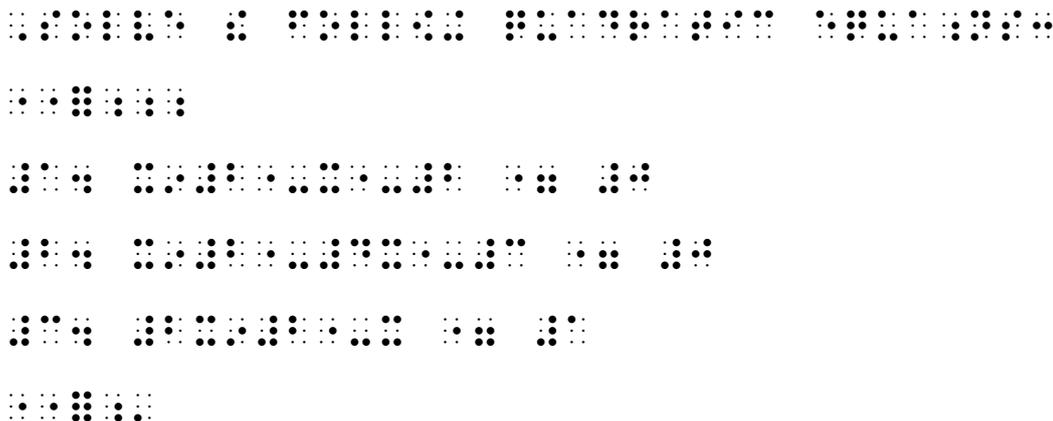
Examples:

Solve the following quadratic equations:

1. $x^2 - x - 2 = 0$

2. $x^2 - 4x - 3 = 0$

3. $2x^2 - x = 1$

**General Note**

Decisions often need to be made in a mathematical context about whether to use:

- a Grade 1 symbol indicator or
- a Grade 1 word indicator or
- a Grade 1 passage indicator with Grade 1 terminator or
- a Grade 1 passage indicator on a line of its own with a Grade 1 passage terminator on a line of its own.

Often there is a choice about Grade 1 indicators in mathematical contexts, with any of the decision options (above) being equally correct. Decisions about option selection are generally associated with user and transcriber preferences, including consideration for simplicity or functionality.

Fractions

Lesson 8 of the *Unified English Braille Training Manual: Introductory Mathematics* introduced how to braille simple fractions, linear fractions and mixed numbers (that is, a whole number followed immediately by a simple fraction).

It is important to understand the definition of a **simple fraction**, that is, what elements and only those elements that can be considered to be a simple fraction, and therefore require use of the simple fraction line, (⠠ dots 34) between the numerator and the denominator.

If a fraction does **not** comply with the **simple numeric fraction** definition (below) for whatever reason, then it will be a **general fraction** and will require a different approach, using different signs.

Simple numeric fraction definition (Revision)

Simple numeric fractions were introduced in Lesson 8 of the *Unified English Braille Training Manual: Introductory Mathematics*.

Definition: A simple numeric fraction is one whose numerator (top of the fraction line) and denominator (bottom of the fraction line) contain only:

- digits,
- decimal points,
- commas, or
- separator spaces,
- **and**, if the fraction line in print (often referred to as the vinculum) is drawn between the two vertically (or near vertically) arranged numbers as shown in the print.

If the fraction complies fully with the definition above for a simple fraction, then a numeric fraction line symbol ($\frac{\cdot}{\cdot}$) should be used between the numerator and the denominator and the numeric fraction line symbol continues the numeric mode and the numeric indicator will not need to be repeated after the fraction line.

Examples:

$\frac{\cdot}{\cdot}$	$\frac{1}{2}$	(vertically) or
$\frac{\cdot}{\cdot}$	$\frac{1}{2}$	(near vertically)
$\frac{\cdot}{\cdot}$	$\frac{196}{28}$	(vertically) or
$\frac{\cdot}{\cdot}$	$\frac{196}{28}$	(near vertically)
$\frac{\cdot}{\cdot}$	$\frac{2.500}{10.000}$	(decimals)
$\frac{\cdot}{\cdot}$	$\frac{10,000}{50,000}$	(commas)
$\frac{\cdot}{\cdot}$	$\frac{10\ 000}{50\ 000}$	(separator spaces)

which Grade 1 indicator is associated with user and transcriber preferences, including consideration for simplicity or functionality.

3. For the purpose of the UEB Online exercises that are associated with this UEB Training Manual: Advanced Mathematics, use the following criteria for implementing Grade 1 mode:
 - Use the Grade 1 **symbol** indicator when there is only one symbol in the sequence requiring a Grade 1 indicator.
 - Use the Grade 1 **word** indicator when there are two or more symbols in the sequence requiring Grade 1 mode, except in a context whereby any literary elements will be affected and as a consequence will also be uncontracted.
 - Use Grade 1 **passage** indicator (with Grade 1 terminator) when a comparison indicator or a space is used in the sequence and Grade 1 indicators are required on both sides of the equation. Remember however, the impact of Grade 1 mode on any literary elements.

Hints:

- Questions 1-5 in Exercise 2 below are general fractions because they contain letters.
- Questions 6-10 in Exercise 2 below are general fractions because they contain something more than digits, commas, decimal points or numeric spaces.
- Remember the numeric indicator (⠠) also sets Grade 1 mode for the remainder of the symbols-sequence.
- When Grade 1 mode is set by the numeric indicator (⠠) it is terminated by a space, hyphen, dash or Grade 1 terminator.

Exercise 2

1. Evaluate $\frac{mn}{3} \div \frac{m}{n}$
2. $\frac{PQR}{XZ}$
3. $\frac{PqR}{xz}$
4. $y = \frac{x}{2}$
5. $\frac{3a}{5} + \frac{a}{5}$
6. $\frac{2^{1/2}}{x+y}$
7. $\frac{8x-24y}{8}$
8. $\frac{4m+12}{3} \times \frac{6m}{m+3}$
9. $\frac{6(h+5)}{h+9}$
10. Simplify $\frac{5w+10}{5}$

Review Exercise 2

1. $\frac{4+3+2}{6-3+8}$
2. $\frac{2/3}{5}$
3. $A = \frac{h}{2}(a+b)$
4. $\frac{3(h+5)}{h+9}$
5. $\frac{5.3}{4,200}$
6. $\frac{\$55}{5}$
7. $\frac{24m}{3cm}$
8. $\frac{3}{10\ 000}$
9. $\frac{4,000}{10}$
10. $\frac{\frac{x}{2} + \frac{y}{3}}{x+y}$

Lesson 3: Operation and Comparison Signs (continued)

Operation Signs

⠠⠠	+	plus (dot 5, dots 2 3 and 5)
⠠⠠	-	minus (dot 5, dots 3 and 6)
⠠⠠	×	times, a multiplication sign as shown in print as a cross (dot 5, dots 2 3 and 6)
⠠⠠	.	a running product sign, shown as a dot, is another means of showing multiplication in print (dot 5, dots 2 and 5 6)
⠠⠠	÷	divided by (dot 5, dots 3 and 4)
⠠⠠	:	ratio sign as shown in print as a colon (dots 2 and 5)
⠠⠠	±	plus or minus (plus over minus)
⠠⠠	∓	minus or plus (minus over plus)

Notes:

1. For the purposes of the UEB Online exercises that are associated with this UEB Training Manual: Advanced Mathematics, always **unspace** the operation sign from the sequence on either side.
2. Although the ratio sign shown above is used to compare two numbers, it is best treated as an operation sign for the purposes of spacing.
3. The ratio sign terminates the effect of the numeric indicator and will therefore need to be repeated before the number that follows.

Examples:

6:12

⠠⠠⠠⠠⠠⠠⠠⠠

2.4.7

⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

Comparison Signs

⠠⠠	=	is equal to
⠠⠠⠠⠠	≠	not equal to (i.e. a line cutting through an equals sign)
⠠⠠	<	is less than

$$30 > 20 > 10$$

⠠⠨⠠⠼⠠⠨⠠⠼⠠⠠

$$-2 \leq x \leq 10$$

⠠⠨⠠⠼⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

$$10 \geq x \geq -2$$

⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

$$3.9 \times 4.1 \approx 16$$

⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

Remember:

As referred to in the General Note in Lesson 1 of this Training Manual, there is often a choice when selecting Grade 1 indicators in mathematical contexts, with any of the options of Grade 1 mode (symbol, word or passage) being equally correct. The choice is associated more with user and transcriber preferences, including consideration for simplicity or functionality.

For the purpose of the exercises in UEB Online that are associated with this UEB Mathematics training program, use the following criteria for implementing Grade 1 mode:

- Use the Grade 1 **symbol** indicator when there is only one symbol in the sequence requiring a Grade 1 indicator.
- Use the Grade 1 **word** indicator when there are two or more symbols in the sequence requiring Grade 1 mode except in a context whereby any literary elements will be affected and as a consequence will also be uncontracted.
- Use Grade 1 **passage** indicator (with Grade 1 terminator) when a comparison indicator or a space is used in the sequence and Grade 1 indicators are required on both sides of the equation. Remember however, the impact of Grade 1 mode on any literary elements.

Exercise 3

1. $16 - 5 \geq 10$
2. $3 + 3 \leq 3 \times 3$
3. $15 < 17$
4. $29 > 23$
5. $a + b \neq c$
6. $15xy \div 3x = \frac{15xy}{3x}$
7. $3(4 - 2x) \geq 18$
8. $2.5 \times 7.7 \approx 19$
9. $x = \pm 15$
10. $3a + 2b = 5c$

Review Exercise 3

1. $5 + -2 =$
2. $9 + -6 = +3$
3. $.672$
4. $\frac{1\ 745\ 711}{6\ 527}$
5. $\frac{\$55.50}{5}$
6. $\frac{20m}{4cm}$
7. $\frac{a}{b+c}$
8. $\frac{x}{4,000}$
9. Expand $(2x - 5)(3x + 7)$
10. $\frac{\frac{p}{5} + \frac{q}{10}}{p+q}$

Lesson 4: Indices

Superscripts and Subscripts

A **superscript** is a distinguishing symbol (such as a numeral or letter) that is written immediately above, OR above and to the right or left of another character.

A **subscript** is a distinguishing symbol (such as a numeral or letter) that is written immediately below, OR below and to the right or left of another character.

Level Change Indicator

The superscript and subscript level change indicators and the braille grouping indicators shown below also have a contracted (Grade 2) meaning and so have been shown with a Grade 1 indicator in front of them. A Grade 1 indicator is used to set Grade 1 mode when the Grade 1 meaning of the symbol could be misread as a contraction meaning.

⠠⠠⠠⠠ Level change up

⠠⠠⠠⠠ Level change down

⠠⠠⠠⠠ Expression directly above (this symbol will be introduced in more detail in the UEB Training Manual: Extension Mathematics).

⠠⠠⠠⠠ Expression directly below (this symbol will be introduced in more detail in UEB Training Manual: Extension Mathematics).

Braille Grouping Indicator

⠠⠠⠠⠠ Open braille grouping indicator (there is no print representation for this symbol in braille)

⠠⠠⠠⠠ Closing braille grouping indicator (there is no print representation for this symbol in braille)

Note:

When both a left-hand subscript and superscript are shown together in print (such as in atomic mass numbers), they are brailled with the subscript first followed by the superscript.

Examples:

x^n

⠠⠠⠠⠠⠠⠠⠠⠠

x_n

⠠⠠⠠⠠⠠⠠⠠⠠

Examples:

x^2

⠠⠠⠠⠠⠠⠠⠠⠠

x^{-2}

⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

OR

⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

Hint:

- The superscript and subscript signs each have a Grade 2 (contracted) meaning so will always require Grade 1 indicators if the sequence is not already shown in Grade 1 mode. For more information about Grade 1 mode, refer to Lesson 1 and Lesson 2 of this Training Manual and also Lesson 3 of the *Unified English Braille Training Manual: Introductory Mathematics*.

Examples:

y^n

⠠⠠⠠⠠⠠⠠⠠⠠

y_n

⠠⠠⠠⠠⠠⠠⠠⠠

x^{-n}

⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

OR

⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

x_{-n}

⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

OR

⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

a^{2n}

⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

OR

⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

x_{2n} 

OR

**Remember:**

As referred to in the General Note in Lesson 1 of this Training Manual, there is often a choice when selecting Grade 1 indicators in mathematical contexts, with any of the options for Grade 1 mode (symbol, word or passage) being equally correct.

However, for the purpose of the UEB Online exercises that are associated with this UEB Training Manual: Advanced Mathematics, use the following criteria for implementing Grade 1 mode:

- Use the Grade 1 **symbol** indicator when there is only one symbol in the sequence requiring a Grade 1 indicator.
- Use the Grade 1 **word** indicator when there are two or more symbols in the sequence requiring Grade 1 mode except in a context whereby any literary elements will be affected and as a consequence will also be uncontracted.
- Use Grade 1 **passage** indicator (with Grade 1 terminator) when a comparison indicator or a space is used in the sequence and Grade 1 indicators are required on both sides of the equation. Remember however, the impact of Grade 1 mode on any literary elements.

Exercise 4

1. 10^2
2. x^{b+1}
3. $x^{\frac{1}{2}}$
4. $x^{1/2}$
5. x_4
6. x_{2+}
7. x^2
8. P_x
9. $a^{-n} = \frac{1}{a^n}$
10. $a^n a^m = a^{n+m}$

Review Exercise 4

1. $0.0025 = 2.5 \times 10^{-3}$
2. x_{a+b}
3. $a_2 + b_3$
4. $x^y z$
5. $\frac{x^{-9}}{4}$
6. $\frac{x^y}{z}$
7. $\frac{x^{1.2}}{4}$
8. $\frac{x^{-\frac{3}{4}}}{4}$
9. $7e_{2x}$
10. $e^{x^3} y$

Exercise 5

1. $\sqrt{25} = 5$
2. $\sqrt{3^2} + 4$
3. $3\sqrt{28} - 2\sqrt{7}$
4. $\sqrt{2t} \times \sqrt{8} = 4\sqrt{3}$
5. $\sqrt{16t} = 4\sqrt{3}$
6. $\frac{6}{2\sqrt{3}}$
7. $5(2\sqrt{3} + 3)$
8. $\sqrt{a^2 + b^2}$
9. $E = \sqrt{1 - \frac{b^2}{a^2}}$
10. $V = \sqrt{2gR}$

Review Exercise 5

1. $x = \sqrt{x^2}$
2. $x = \frac{(5 + \sqrt{45})}{2}$
3. $(a - 4)^2$
4. $(b - 4) = \pm\sqrt{3}$
5. $-(5x - 3) \geq 2$
6. $F = \frac{mv^2}{r}$
7. $m = \frac{y_2 - y_1}{x_2 - x_1}$
8. 10^{-4}
9. $(2r)^{\frac{1}{2}}$
10. $T_n = a + (n - 1)d$

Lesson 6: Shape Indicators (continued) and Miscellaneous Symbols

Shape Indicators

The shape indicators shown below also have a contracted (Grade 2) meaning and so have been shown with a Grade 1 indicator in front of them. A Grade 1 indicator is used to set Grade 1 mode when the Grade 1 meaning of the symbol could be misread as a contraction meaning and Grade 1 mode has not already been established in the sequence. For more information about Grade 1 mode, refer to Lesson 1 of this Training Manual and also Lesson 3 of *Unified English Braille Training Manual: Introductory Mathematics*.

⠠⠠		shape indicator
⠠⠠		shape termination indicator (there is no print representation for this braille symbol)
⠠⠠⠠⠠	□	square (as introduced in Lesson 6, <i>Unified English Braille Training Manual: Introductory Mathematics</i>)
⠠⠠⠠⠠	△	triangle, equilateral (as introduced in Lesson 6, <i>Unified English Braille Training Manual: Introductory Mathematics</i>)
⠠⠠⠠⠠	⬠	pentagon
⠠⠠⠠⠠	⬡	hexagon
⠠⠠⠠⠠	⬢	octagon
⠠⠠⠠⠠⠠	▭	parallelogram
⠠⠠⠠	○	circle (as introduced in Lesson 6, <i>Unified English Braille Training Manual: Introductory Mathematics</i>)

Transcriber defined shapes

⠠⠠	transcriber-assigned shape indicator
----	--------------------------------------

Note:

A transcriber-assigned shape indicator should not be used if the print symbol has already an assigned braille sign. The indicator should precede a short series of initials or a single Grade 1 word. The definitions of all the transcriber-assigned shape indicators used in the transcription should be available to the reader in either a transcriber's note or on a special page.

⠐⠐⠐	≡	is congruent/equivalent to (three horizontal lines)
⠐⠐⠐⠐		is similar to (three vertical lines)
⠐⠐⠐	[open square bracket
⠐⠐⠐]	close square bracket
⠐⠐⠐	{	open curly bracket
⠐⠐⠐	}	close curly bracket
⠐		dot 5 continuation indicator (used when the braille sequence is too long for the line and needs to be broken)
⠐⠐		visible blank space (an omission, which often occurs in fractions)
⠐⠐	/	cancelling sign
⠐⠐	.	recurring decimal (dot over the top of a number)
⠐⠐	✓	tick sign (not to be confused with the root sign)

Notes:

1. In general, the spacing of symbols follows the print.
2. Some of the signs listed above have a Grade 2 (contracted) meaning. A Grade 1 indicator will therefore be necessary if the sequence is not already in Grade 1 mode.
3. If the braille representation for a print sequence does not fit on the line, then a dot 5 continuation indicator placed at a logical place immediately following the last character may be required to show the braille is continuing for the remainder of the print sequence. Usually the preferred place to break is before a comparison sign or an operation sign.
4. Braille grouping signs are needed for the recurring decimal to explicitly show the character that has the dot above.
5. Remember when both a subscript and superscript are shown together in print, they are brailled with the subscript first followed by the superscript.

Examples:

6'3''

⠐⠐⠐⠐⠐⠐⠐⠐⠐⠐⠐⠐⠐

 $\frac{2}{8} = \frac{1}{4}$

$$4! = 4 \cdot 3 \cdot 2 \cdot 1$$

OR

$$4! = 4 \cdot 3 \cdot 2 \cdot 1$$

4! = 4 · 3 · 2 · 1 (shown using a running product)

$$4! = 4 \cdot 3 \cdot 2 \cdot 1$$

$$\therefore x = 2$$

$$\therefore x = 2$$

$$\angle A + \angle B = 90^\circ$$

$$\angle A + \angle B = 90^\circ$$

$$AB \parallel CD$$

$$AB \parallel CD$$

$$AB \perp CD$$

$$AB \perp CD$$

$$\triangle ABC \cong \triangle DEF$$

$$\triangle ABC \cong \triangle DEF$$

OR

$$\triangle ABC \cong \triangle DEF$$

$$\triangle ABC \equiv \triangle DEF$$

$$\triangle ABC \equiv \triangle DEF$$

OR

$$\triangle ABC \equiv \triangle DEF$$

$$\frac{6 \times 3}{12} = \frac{\phi \times 3}{\phi \times 2}$$

$$\frac{6 \times 3}{12} = \frac{\phi \times 3}{\phi \times 2}$$

$$\frac{6 \times 3}{12} = \frac{\phi \times 3}{\phi \times 2}$$

OR

Note the use of the dot 5 continuation indicator in the following example which may not be necessary when the = sign is taking the whole of the expression to the next line.

Exercise 6

1. $0.35\dot{1}2\dot{3}$
2. $\angle EFG$ is adjacent to $\angle GFH$
3. $\triangle ABC \cong \triangle DEF$
4. $\angle OCA \neq \angle OCB$
5. $3(8 + 5) = 3 \times \square + 3 \times \square$
6. $EF \parallel GH$
7. $\therefore PQ \perp RS$
8. {Craig, Frances, Tarna}
9. $\angle ABC = \angle DEF$
10. $\triangle EDF \parallel \triangle FDG$

Review Exercise 6

1. $(x + 5)^2 = x^2 + 10x + 25$
2. $P(x) = 5 - 3x + x^2$
3. $(9y + 1)(7y + 2) = 0$
4. $v = \sqrt{\frac{g}{k}}$
5. $A = \frac{1}{2}h(a + b)$
6. $S = \frac{n}{2}(a + l)$
7. $\frac{2y}{5} = 11 - \frac{y}{3}$
8. $T_n = ar^{n-1}$
9. $\sqrt{8.41 \times 10^{-8}}$
10. $(xy)^3$

Lesson 7: Functions

Trigonometric functions

Common trigonometric functions are Sine, Cosine and Tangent. These functions are usually abbreviated in print as sin, cos and tan.

1. Sine (Sin), Cosine and Tangent may be contracted unless already in Grade 1 mode.
2. Where the function name is **preceded** or **followed** by a lowercase letter, a space may be needed between the letter and the function to remove any ambiguity as to where the function name begins and ends.

Examples:

⠠⠠⠠⠠ ⠠⠠⠠⠠ sinx

⠠⠠⠠⠠⠠⠠ ⠠⠠⠠⠠ cosy

⠠⠠⠠⠠ ⠠⠠⠠⠠ ⠠⠠⠠⠠ xsiny

If the function name is directly preceded or followed by a number, then the number should be written **unspaced** from the function name.

Remember the numeric indicator (⠠⠠) also sets Grade 1 mode for the next symbols-sequence.

Examples:

⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠ ⠠⠠tan45°

⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠ ⠠⠠7sin45°

⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠ sin45

⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠ xtan60

⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠ xsin60

A space is not needed if the function name is already separated by a bracket or by a braille indicator, such as a capitalisation indicator, a fraction indicator, or a Greek letter indicator, which is explained more fully in Lesson 8 of this Training Manual.

Remember the opening and closing fraction indicators have a Grade 2 (contracted) meaning so unless you are already in Grade 1 mode, a Grade 1 symbol indicator will be required before the opening and closing fraction indicator.

Exercise 7

1. $\sin A$
2. $\log(x + y)$
3. $6\tan 90^\circ$
4. $\sin 30$
5. $5\sin 45$
6. $7\cos 5x$
7. $\log a + \log b = \log ab$
8. $\log_4 = -2$
9. $\frac{\sin C}{c} = \frac{\sin A}{a}$
10. $\frac{\sin Z}{35} = \frac{\sin 70^\circ}{7}$

Review Exercise 7

1. $\frac{5}{\square} = \frac{30}{36}$
2. In $\triangle DEF$, $\angle E = 90^\circ$
3. $\sqrt{9a^8}$
4. $\left(\frac{25}{49}\right)^{\frac{1}{2}}$
5. $\sqrt{m^2}$
6. $\frac{18 \cos 12^\circ}{13 \tan 68^\circ}$
7. $\sqrt[6]{101.9}$
8. In $\triangle KLM$, $\angle M = 27^\circ 51'$
9. $\sin Z = \frac{35 \sin 70^\circ}{45}$
10. $\log_e\left(\frac{2}{3}\right)$

Lesson 8: Greek Letters

Greek letters are used extensively in Mathematics. While only a small number have been used in this Training Manual, the principles for use remain the same for all. Refer to the “Unified English Braille Guidelines for Technical Material” (International Council on English Braille, 2014) for a complete list.

⠠⠠	α	alpha (lower case)	⠠⠠⠠	A	Capital
⠠⠠	β	beta (lower case)	⠠⠠⠠	B	Capital
⠠⠠	δ	delta (lower case)	⠠⠠⠠	Δ	Capital
⠠⠠	ε	epsilon (lower case)	⠠⠠⠠	E	Capital
⠠⠠	γ	gamma (lower case)	⠠⠠⠠	Γ	Capital
⠠⠠	θ	theta (lower case)	⠠⠠⠠	θ	Capital
⠠⠠	λ	lambda (lower case)	⠠⠠⠠	Λ	Capital
⠠⠠	μ	mu (lower case)	⠠⠠⠠	M	Capital
⠠⠠	π	pi (lower case)	⠠⠠⠠	Π	Capital
⠠⠠	σ	sigma (lower case)	⠠⠠⠠	Σ	Capital

Remember:

As referred to in the General Note in Lesson 1, there is often a choice of Grade 1 indicator, with any of the options of Grade 1 mode (symbol, word or passage) being equally correct. However, for the purpose of the UEB Online exercises associated with this training manual, use the following criteria for implementing Grade 1 mode:

- Use the Grade 1 **symbol** indicator when there is only one symbol in the sequence requiring a Grade 1 indicator.
- Use the Grade 1 **word** indicator when there are two or more symbols in the sequence requiring Grade 1 mode except in a context whereby any literary elements will be affected and as a consequence will also be uncontracted.
- Use Grade 1 **passage** indicator (with Grade 1 terminator) when a comparison indicator or a space is used in the sequence and Grade 1 indicators are required on both sides of the equation. Remember however, the impact of Grade 1 mode on any literary elements.

Exercise 8

1. $A = \frac{\theta}{360} \times 2\pi r$
2. $V = \frac{1}{3} \pi r^2 h$
3. $A = 2\pi r^2 + 2\pi r h$
4. $S = \theta r(r + l)$
5. $\therefore \Sigma x = 195$
6. $V = \frac{1}{3} \pi$
7. $C = 2\pi r$
8. Standard deviation (σ_n)
9. Mean = $\frac{\Sigma fx}{\Sigma f}$
10. surface area = $\pi r s + \pi r^2$

Review Exercise 8

1. $0 \leq x \leq 2\pi$
2. $\frac{25\pi}{8} m^2$
3. $(\log_e x)^4$
4. $\sin\theta = \frac{14.6 \sin 48^\circ}{12.6}$
5. $\angle DAC = \angle ACB$
6. $\therefore \triangle ABC \equiv \triangle ABD$
7. $\sqrt[3]{8u^{18}}$
8. $25gh \div \square = 5g$
9. $y = -\frac{1}{2}(x + 4)(2 - x)$
10. $DE \parallel AC$ and $CE:EB = 2:3$

Lesson 9: Review Test

Congratulations on reaching this Review lesson. The content of each lesson has built upon preceding lessons, with the overall structure of this Training Manual designed to reinforce several foundational principles, including the following:

1. The numeric indicator (⠼) sets numeric mode **and** Grade 1 mode for the remainder of the symbols-sequence.
2. Numeric mode is transitive over the 10 digits, the full stop, the comma, the numeric space, the simple fraction line and the continuation indicator and is terminated by symbols such as the hyphen, dash, slash/oblique stroke and colon whereby the numeric indicator will need to be repeated.
3. When Grade 1 mode is established by the numeric indicator, it is terminated by a space, hyphen, dash and Grade 1 terminator.
4. A braille symbol may have both an uncontracted (Grade 1) meaning and a contracted (Grade 2) meaning. A Grade 1 indicator is used to set Grade 1 mode when the Grade 1 meaning of a symbol could be misread as a contraction meaning or a numeric meaning.
5. Understanding of when a fraction is considered a “simple fraction”. If the sequence does not satisfy the definition of a simple fraction, then it must be treated as a general fraction.
6. The rules associated with “the next item” should be considered and are particularly relevant in the use of subscripts and superscripts.

Remember:

As referred to in the General Note in Lesson 1 of the UEB Training Manual: Advanced Mathematics and throughout the preceding lessons, the choice of options for Grade 1 mode (symbol, word or passage) are equally correct in mathematical contexts. However, for the purpose of completing the following Review Test in the UEB Online course, use the following criteria for implementing Grade 1 mode:

- Use the Grade 1 **symbol** indicator when there is only one symbol in the sequence requiring a Grade 1 indicator.
- Use the Grade 1 **word** indicator when there are two or more symbols in the sequence requiring Grade 1 mode except in a context whereby any literary elements will be affected and as a consequence will also be uncontracted.
- Use Grade 1 **passage** indicator (with Grade 1 terminator) when a comparison indicator or a space is used in the sequence and Grade 1 indicators are required on both sides of the equation. Remember however, the impact of Grade 1 mode on any literary elements.

Review Test

1. $A = \pi r^2$
2. $\frac{4\sqrt{3} \times \sqrt{18}}{\sqrt{12}}$
3. $time = \frac{distance}{speed}$
4. $4r^4 \div 2r$
5. $x^{7.9}$
6. $A = P(1 + r)^n$
7. $3\sqrt{8}$
8. x^{-2}
9. x^3y
10. $(2x + 9)^2$
11. $\frac{x^4}{4}$
12. $\sqrt[3]{2x}$
13. $\{2, 4, -6\}$
14. $P(E) = \frac{n(E)}{n(S)}$
15. $\sqrt{96} \div \sqrt{12}$
16. $y = -\frac{1}{2}(x - 1)^5$
17. $AB \parallel PQ$
18. $\therefore \triangle MNO \cong \triangle PQR$
19. $3! = 3 \cdot 2 \cdot 1$
20. $\frac{3x^2 - 6x}{x^2 + x - 6}$

Lesson 10: Advanced Test

Congratulations on reaching this Advanced Test. The content of each lesson has built upon preceding lessons, with the overall structure of this Training Manual designed to reinforce several foundational principles, including the following:

1. The numeric indicator (⠠) sets numeric mode **and** Grade 1 mode for the remainder of the symbols-sequence.
2. Numeric mode is transitive over the 10 digits, the full stop, the comma, the numeric space, the simple fraction line and the continuation indicator and is terminated by symbols such as the hyphen, dash, slash/oblique stroke and colon whereby the numeric indicator will need to be repeated.
3. When Grade 1 mode is established by the numeric indicator, it is terminated by a space, hyphen, dash and Grade 1 terminator.
4. A braille symbol may have both an uncontracted (Grade 1) meaning and a contracted (Grade 2) meaning. A Grade 1 indicator is used to set Grade 1 mode when the Grade 1 meaning of a symbol could be misread as a contraction meaning or a numeric meaning.
5. Understanding of when a fraction is considered a “simple fraction”. If the sequence does not satisfy the definition of a simple fraction, then it must be treated as a general fraction.
6. The rules associated with “the next item” should be considered, and are particularly relevant in the use of subscripts and superscripts.

Remember:

As referred to in the General Note in Lesson 1 of the UEB Training Manual: Advanced Mathematics, and throughout the preceding lessons, the choice of options for Grade 1 mode (symbol, word or passage) are equally correct in mathematical contexts. However, for the purpose of completing the following Advanced Test in the UEB Online course, use the following criteria for implementing Grade 1 mode:

- Use the Grade 1 **symbol** indicator when there is only one symbol in the sequence requiring a Grade 1 indicator.
- Use the Grade 1 **word** indicator when there are two or more symbols in the sequence requiring Grade 1 mode except in a context whereby any literary elements will be affected and as a consequence will also be uncontracted.
- Use Grade 1 **passage** indicator (with Grade 1 terminator) when a comparison indicator or a space is used in the sequence and Grade 1 indicators are required on both sides of the equation. Remember however, the impact of Grade 1 mode on any literary elements.

Advanced Test

1. $\sqrt{a^2 + b^2}$
2. $\sqrt[3.4]{8}$
3. The mean = $\frac{\Sigma fx}{\Sigma f}$
4. $y = x^4 - x^3 - 10x^2 - 8x$
5. $\log \frac{1+x}{x}$
6. $\frac{a}{\sin 20^\circ} = \frac{12}{\sin 60^\circ}$
7. $y = \log_a f(x)$
8. $f(L) = 2\pi \sqrt{\frac{L}{g}}$
9. $f(x) = 2 \cos \left(x + \frac{\pi}{2}\right) + 1$
10. $\log(x^2 - x - 2) - \log(x + 1)$
11. $\log \sqrt{x^2 - 4x + 4}$
12. $\log_a \frac{x}{\sqrt{y}}$
13. $\sin 120^\circ = 0.87$
14. $\tan \theta = \frac{\sin \theta}{\cos \theta}$
15. $\cos \frac{\pi}{5} = \frac{1 + \sqrt{5}}{4}$
16. $\log_e x$
17. $m = \frac{y_2 - y_1}{x_2 - x_1}$
18. $\frac{1}{k(k+1)(k+2)}$
19. $\frac{4.1 \times \sqrt{48.12}}{26.23}$
20. $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

References

Howse, J. (2018). *Unified English Braille Maths K – 6*. Workshop presented at RIDBC Renwick Centre, North Rocks, NSW.

Howse, J. (2018). *Unified English Braille Maths 7 – 12*. Workshop presented at RIDBC Renwick Centre, North Rocks, NSW.

Howse, J. Riessen, K. and Holloway, L. (Eds.). (2016). *Unified English Braille: Australian Training Manual*. Available from <http://brailleaustralia.org/unified-english-braille/unified-english-braille-australian-training-manual-2013/>

International Council on English Braille. (2014). *Unified English Braille Guidelines for Technical Material*. Available from http://www.iceb.org/guidelines_for_technical_material_2014.pdf

Simpson, C. (Ed.). (2013). *The Rules of Unified English Braille* (2nd ed.).

International Council on English Braille (ICEB). Available from [http://iceb.org/Rules of Unified English Braille 2013.pdf](http://iceb.org/Rules_of_Unified_Engish_Braille_2013.pdf)